Applied ARCHITECTURAL ACOUSTICS

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Sound—the echo of command that bid the stars to shine

FOREWORD

Notable progress has been made during the past two decades in the acoustical design of sound recording and reproducing rooms. This progress is described in the following pages. But this book is more than a progress report of work done by others; it presents the physical and mathematical reasoning—in some instances for the first time—for much of this progress. Although most of this reasoning is based on *geometrical* rather than *physical* (wave) acoustics, the author is aware of, or actually anticipates, the effects of the more exact analyses of wave acoustics. This book not only helps in preparing the way for the advent of wave acoustics in studio and theater design, but, what is more important, it is thoroughly practical in specifying (1) the optimal size and shape of sound studios and motion picture theaters, and (2) the insulative, absorptive, and reflective materials and constructions for optimal acoustics in such rooms.

VERN O. KNUDSEN

INTRODUCTION

Room acoustics has come to constitute an important component in modern architecture. Functional building design is incomplete without it. Planning for good hearing conditions is no less important than planning for good illumination, or for comfort, or convenience, whatever the case may be.

Permitting the marring of music or the spoken word by excessive room reverberation, echoes, noise, etc., is no less excusable than providing inadequate ventilation or heating conditions in a building. The ear, like the eye, was made to respond to vital stimuli—social, economic, and cultural—to a lecture, a message, or a song. To deprive the ear of this signal perception is to deprive it of its life.

Architectural acoustics is not purely functional. It combines the scientific with the artistic. It provides the architect with forms and treatment of high esthetic appeal. Today it is no longer necessary to copy buildings of reputedly good hearing conditions; it is no longer necessary to be tradition-bound. Great flexibility exists in achieving identical results by dissimilar methods, a flexibility which will become greater still as new acoustic materials are developed and novel methods of construction invented.

This book is directed to architects, engineers, contractors, and all those connected with the planning and the construction of buildings in which acoustics has been given preference. It is not a text-book for colleges. It is not fundamentally analytical. A handbook for the man in the field, it was kept practical, without, however, shunning important theoretical matters.

In this book special attention is given to the acoustics of rooms in which sound is either recorded, such as motion picture studios or broadcasting stations, or is reproduced electrically, such as motion picture theaters or monitoring rooms. The reason for this emphasis is twofold: 1. in such enclosures satisfactory acoustics is of prime importance, the acoustic treatment of these rooms illustrating the significant features necessary for desirable listening conditions; 2. whatever information on this important subject has been printed, is scattered in an apparently haphazard manner through a number of journals and trade-magazines and a few—very few—books. Nevertheless, the acoustic treatment of several other types of rooms has been included, to make the text comprehensive.

Architectural acoustics is still not taught as extensively in our technical schools as it deserves, with the result that motion picture theaters are still being built with concave, insufficiently treated rear walls; hospitals are noisy and class-rooms echoic. But with the increasing mechanization of our life, such matters will perforce demand the attention of a larger number of architects and technicians. Buildings will be constructed which will be a home for both the body and the mind.

This new science is extremely interesting. The newcomer to the field will soon learn that rooms are like people; that no two of them are alike. A room is not a logical unit like a spur gear or a brake-shoe. It mirrors some of the individuality of the designer and of the men that plastered the walls and laid the floor. A convex wood splay can be made sonorous, attractive and durable, and it can be made rattly and to show ugly fissures, nailheads, and scaly paint. A floor can be built which does not squeak or carry footfalls, and plumbing can be installed which does not knock and hammer. To do a job well demands that one understands the reasons behind the various requirements and manipulations. It will not do to scale down an acoustically satisfactory enclosure in the hope that the miniature will provide equally desirable results. Each room must be analyzed by itself, and its construction supervised with care and deliberation.

It is not always the privilege of the architect and the engineer to build "from the ground on up." Often he has to do an alteration job or add a unit to an existing structure. The writer once was requested to design a scoring stage on top of another building, which thus practically fixed the width and length of the stage. The finished studio was quite satisfactory, and is still in operation. Mastery of a complex subject requires a knowledge of the elements of the subject. To this extent, this book provides the alphabet of architectural acoustics.

After the pioneer work of W. C. Sabine, near the turn of the century, a powerful impetus was given to architectural acoustics by the introduction of sound to motion pictures. Almost overnight, extensive research and development work on the subject came into existence. There was a time when more was known about sound in Hollywood than in the rest of the country. The physics department of the University of California at Los Angeles, under the able leadership of Dr. Vern O. Knudsen, became the Mecca for information on acoustics. Among the graduate students of this school at the time was a young chap, the author, who threw himself whole-heartedly into the new science, and was soon able to extend progress in the field. As time went on, his work concerned itself with buildings outside the motion picture industry, and included radio stations, churches, theaters, etc.

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Chapter I

FUNDAMENTAL EQUATIONS AND DEFINITION OF SYMBOLS

Sound, like many other forms of energy, is a wave phenomenon. In acoustics many practical problems can be solved only by accepting the concept of wave motion, in spite of the wide applicability of purely geometric methods. The situation here is not far different from that in optics, although no *corpuscular theory* has been established in the field of sound. Sound and light have only some of their mathematics in common; their natures are entirely different.

Wave Motion

Wave motion may be defined as the progressive disturbance propagated in a medium by the periodic vibration of the particles of the medium. A complete set of the recurrent values of such a periodic disturbance is called a *cycle*. The number of cycles per unit time is the *frequency*, the inverse of which is the *period*.

The most important type of wave motion is the simple harmonic or sinusoidal motion. Figure 1 symbolizes such a wave, which mathematically can be expressed as

$$\begin{aligned} y &= A \sin \frac{2\pi}{\lambda}(ct - x) \\ &= A \sin 2\pi f\left(t - \frac{x}{c}\right) \\ &= A \sin(\omega t - kx) \\ &= A \sin(a - bx) \end{aligned}$$

where c = the velocity of propagation of the wave

$$\lambda = \text{wave-length}$$

$$f = \text{frequency} = \frac{c}{\lambda}$$

$$\omega = 2\pi f$$

$$k = \frac{2\pi f}{c} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\frac{a}{b} = ct = \text{phase}$$

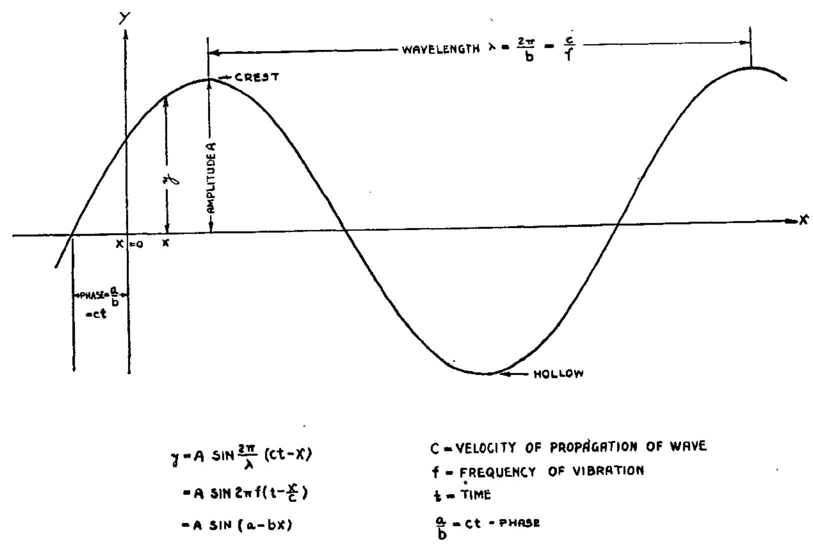

Figure 1

SINE WAVE

The above equation indicates a *plane wave* because, in a three-dimensional model of the wave, all points in a plane perpendicular to the x-axis are in the same phase. A plane wave may also be described as one the wavefront of which is a plane. Similarly, a *spherical wave* is one the wavefront of which is a sphere. A *cylindrical wave* is one the wavefront of which is a cylinder. Another way of differentiating between plane, spherical, and cylindrical waves would be to say that the origin of each wave is a plane, a point, and a cylinder, respectively.

Transverse wave motion is that in which the vibration of the particles is perpendicular to the direction of propagation. As an example of transverse wave motion we may mention the wave motion along a stretched string when plucked. *Longitudinal* wave motion is that in which the vibration of the particles is parallel to the direction of propagation.

Particle velocity is defined as the instantaneous velocity of a given infinitesimal part of the medium, with reference to the medium as a whole, due to the passage of the wave. It should not be confused with the *velocity of propagation,* or *speed,* of the wave, which is the time rate of change with which the disturbance is propagated in the medium.

A *complex wave,* if periodic, is obtained by the algebraic superposition of several sinusoidal waves.

Sound

Sound is an alteration in pressure, particle displacement, or particle velocity, propagated in an elastic material, or the superposition of such propagated alterations. In an extended sense, the alterations may include other physical quantities, such as density or temperature.

Sound is also the sensation produced upon the ear by the compressional disturbances mentioned.

Conventionally and conveniently, sound as used in acoustics has come to mean any vibratory motion of bodies, the transmission of these vibrations in a solid or fluid medium, and the sensation produced by the communication of the vibrations to the human auditory mechanism.

When the variation in pressure, particle displacement, or particle velocity is a simple sinusoidal function of time, the resultant sound is designated a *pure tone.* A pure tone can be described quantitatively by the frequency and the amplitude of its vibration. In the case of a plane wave in a gaseous medium, this may be done by writing

$$d = D_m \sin \omega t \qquad \text{(cm)}$$
$$v = V_m \cos \omega t \qquad \text{(cm/sec)}$$
$$= \omega D_m \cos \omega t$$
$$b = -B_m \sin \omega t \qquad \text{(cm/sec}^2\text{)}$$
$$= \omega^2 D_m \sin \omega t$$
$$p = P_m \cos \omega t \qquad \text{(dynes/cm}^2\text{)}$$
$$= \rho c \omega D_m \cos \omega t$$

where d, v, b, p = instantaneous particle displacement, particle velocity, particle acceleration, and pressure respectively

D_m, V_m, B_m, P_m = maximum values of particle displacement, particle velocity, particle acceleration and pressure respectively

ρ = density of the gaseous medium

The above equations show that in a plane sound wave the particle velocity and pressure are in phase. This means that when the pressure is a maximum the particle velocity is a maximum, and when the pressure is a minimum the particle velocity is a minimum. Similarly, the particle displacement and the particle acceleration are said to be in phase. This is not true, however, of a spherical wave.

It appears desirable, at this point, to explain a certain characteristic of sound pressure and particle velocity to facilitate understanding of these two important physical properties. Sound pressure is a scalar, meaning a quantity with magnitude but no direction. For ready identification, we may mention such scalars as mass, density and temperature. Particle velocity is a vector, a quantity which has both magnitude and direction. Therefore, when the particle velocity of a plane progressive wave is measured, the value of the velocity will be found to vary with direction, being a maximum along the line of propagation of the sound.

Three other physical quantities are frequently employed in the evaluation of sound fields—intensity, energy density, and power. The *intensity* of a sound field in a specified direction at a point is the sound energy transmitted per unit time in the specified direction, through unit area normal to this direction at that point. The absolute unit is the erg per second per square centimeter, but sound intensity usually is expressed in watts per square centimeter. In the case of a plane or spherical free * wave having the effective (root-mean-square) sound pressure, P, and velocity of propagation, c (centimeters per second), in a medium of density ρ (grams per cubic centimeter), the intensity in the direction of propagation is given by

$$I = \frac{P^2}{\rho c}\left(\frac{\text{ergs}}{\text{cm}^2\text{sec}}\right)$$

For air at 20° C and 76 cm of mercury, with c equal to 34340 centimeters per second and ρ equal to 0.001205 gram per cubic centimeter, the intensity becomes

$$I = \frac{P^2}{41.4}\left(\frac{\text{ergs}}{\text{cm}^2\text{sec}}\right)$$

$$= \frac{P^2}{414}\left(\frac{\text{microwatts}}{\text{cm}^2}\right)$$

Sound energy is usually expressed as energy per unit volume, or *energy density,* the unit of which is the erg per cubic centimeter.

Sound-energy flux is the rate of flow of sound energy over one period through any specified area. The unit is the erg per second.

Power is defined as the time rate at which work is done. The unit is the erg per second. The *watt* is equal to one joule (ten million ergs) per second. The *mean power* radiated by a source of plane waves of area S (large compared to λ^2) and uniform in amplitude is expressed as

$$W = \tfrac{1}{2}\rho c\omega^2 D_m{}^2 S$$
$$= \tfrac{1}{2}\rho c S V_m{}^2$$

* Unencumbered by interference effects.

Figure 2 shows the relationship between the various quantities defined above. It refers to maximum values.

	D	P	V	B	I	E	W
D		$\frac{P}{\rho\omega c}$	$\frac{V}{\omega}$	$\frac{B}{\omega^2}$	$\frac{1}{\omega}\sqrt{\frac{2I}{\rho c}}$	$\frac{1}{\omega}\sqrt{\frac{2E}{\rho}}$	$\frac{1}{\omega}\sqrt{\frac{2W}{\rho cS}}$
P	$\rho c\omega D$		ρcV	$\frac{\rho cB}{\omega}$	$\sqrt{2\rho cI}$	$\sqrt{2\rho c^2 E}$	$\sqrt{\frac{2\rho cW}{S}}$
V	ωD	$\frac{P}{\rho c}$		$\frac{B}{\omega}$	$\sqrt{\frac{2I}{\rho c}}$	$\sqrt{\frac{2E}{\rho}}$	$\sqrt{\frac{2W}{\rho cS}}$
B	$\omega^2 D$	$\frac{\omega P}{\rho c}$	ωV		$\omega\sqrt{\frac{2I}{\rho c}}$	$\omega\sqrt{\frac{2E}{\rho}}$	$\omega\sqrt{\frac{2W}{\rho cS}}$
I	$\frac{1}{2}\rho c\omega^2 D^2$	$\frac{P^2}{2\rho c}$	$\frac{\rho cV^2}{2}$	$\frac{\rho cB^2}{2\omega^2}$		Ec	$\frac{W}{S}$
E	$\frac{1}{2}\rho\omega^2 D^2$	$\frac{P^2}{2\rho c^2}$	$\frac{1}{2}\rho V^2$	$\frac{\rho B^2}{2\omega^2}$	$\frac{I}{c}$		$\frac{W}{cS}$
W	$\frac{1}{2}\rho c\omega^2 D^2 S$	$\frac{P^2 S}{2\rho c}$	$\frac{1}{2}\rho cV^2 S$	$\frac{\rho cB^2 S}{2\omega^2}$	IS	EcS	

D = DISPLACEMENT (CM)
V = PARTICLE VELOCITY (CM/SEC)
P = SOUND PRESSURE (DYNES/CM²)
B = ACCELERATION (CM/SEC²)
I = SOUND INTENSITY (ERGS/CM² SEC)
E = ENERGY DENSITY (ERGS/CM³)
W = POWER (ERGS/SEC)
ρ = DENSITY OF MEDIUM (GR/CM³)
f = FREQUENCY (CYCLES/SEC)
S = AREA (CM²)
c = SPEED OF SOUND (CM/SEC)
$\omega = 2\pi f$

$P_i = \rho\omega cD \sin k(ct-x) = P \sin k(ct-x)$ $\quad V_i = \omega D \sin k(ct-x) = V \sin k(ct-x)$ $\quad B_i = \omega^2 D \cos k(ct-x) = B \cos k(ct-x)$

Figure 2

FUNDAMENTAL RELATIONSHIPS OF A PLANE PROGRESSIVE SOUND WAVE (*Maximum Values*)

The relationship between maximum and root-mean-square values of a sine wave is as follows

$$P = \frac{P_m}{\sqrt{2}}$$

where P is the root-mean-square and P_m the maximum value.

While many acoustic measurements deal with finding the absolute value of the pressure, particle velocity, etc., others concern themselves with establishing their relative values. For this purpose so-called *level* measurements are made, which are always based on a reference value and commonly expressed in decibels.

In particular, the *pressure-level,* in decibels, of a sound is twenty times the logarithm to the base 10 of the ratio of the root-mean-square pressure, P, of this sound to the reference pressure. The root-mean-square *reference sound pressure,* P_o, is 0.0002 dyne per square centimeter, which is the value of the pressure near the threshold of audibility for a 1,000-cycle pure tone.

The *velocity-level,* in decibels, of a sound wave is twenty times the logarithm to the base 10 of the ratio of the root-mean-square particle velocity, V, of the wave to the reference velocity. The root-mean-square *reference particle velocity,* V_o, is 5×10^{-6} centimeter per second.

The *intensity-level,* in decibels, of a sound is 10 times the logarithm to the base 10 of the ratio of the intensity of this sound to the reference intensity. The *reference intensity,* I_o, is 10^{-16} watt per square centimeter, which is the value of the intensity near the threshold of audibility for a 1,000-cycle pure tone.

The relationship between the various levels defined above is as follows

$$20 \log \frac{P}{P_0} = 20 \log \frac{V}{V_0} = 10 \log \frac{I}{I_0}$$

On this scale

zero decibels = 0.0002 dyne per square centimeter

$\simeq 5 \times 10^{-6}$ centimeter per second

$\cong 10^{-16}$ watt per square centimeter

$\cong$ 73.8 decibels below 1 dyne per square centimeter

$\cong$ 93.8 decibels below 10 dynes per square centimeter.

The *sensation-level* at a given frequency is the number of decibels by which the sound exceeds the threshold of hearing at that frequency. It should be noted that the reference pressure, reference particle velocity, and reference intensity in the case of sensation-level is a function of frequency, so that it becomes important to note what is meant by zero decibels. Figure 3 shows the threshold of hearing for a normal person, and it is seen for instance, that at 90 cycles (reference pressure = 0.02 dynes per square centimeter) a sensation-level of 60 decibels refers to a

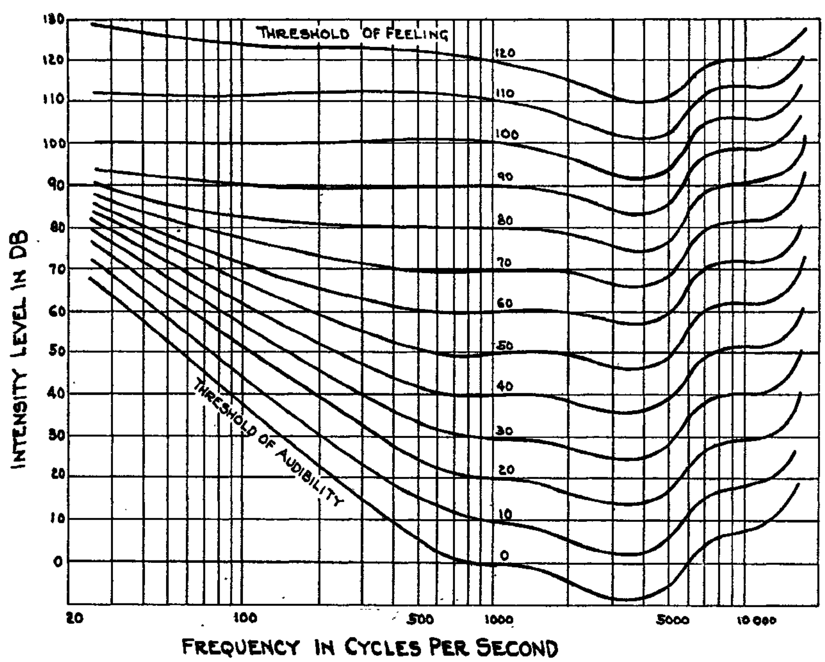

Figure 3

LOUDNESS CONTOURS

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sound pressure of 20 dynes per square centimeter. At 1,000 cycles, sensation-level and intensity-level are equal, and a sensation-level or intensity-level of 60 decibels refers to a sound pressure of 0.2 dynes per square centimeter.

Loudness-level of a sound is the intensity-level of a 1,000-cycle tone which sounds equally loud. The unit of loudness-level is the phon. Again, at 1,000 cycles, loudness-level (in phons) and intensity-level are equivalent.

When sound in its progress comes to the boundary of a different medium, part of the sound energy is reflected and part of it is transmitted into the new medium. By definition, the acoustic *reflectivity* of a surface not a generator, is the ratio of the rate of flow of sound energy reflected from the surface (on the side of incidence) to the incident rate of flow. Unless otherwise specified, all possible directions of incident flow are assumed to be equally probable. Also, unless otherwise stated, the values given apply to a portion of an infinite surface, thus eliminating edge effects. The acoustic *absorptivity* of a surface is equal to one minus the reflectivity of that surface. The acoustic *transmissivity* of an interface is the ratio of the rate of flow of transmitted sound energy to the rate of incident flow. Again, unless otherwise specified, all directions of incident flow are assumed to be equally probable.

The *sabin* is the unit of equivalent absorption; it is equal to the equivalent absorption of 1 square foot of a surface of unit absorptivity, i.e., of 1 square foot of surface which absorbs all incident sound energy (such as 1 square foot of open-window *may* do).

CHAPTER II

GEOMETRIC ACOUSTICS

For the study of sound in rooms, geometric acoustics represents a highly effective instrument, subject only to the three limitations noted below. By charting sound rays, and observing their paths and spatial relationships, many a problem in architectural acoustics can be solved in comparatively short order. The method, elementary in its visual representation, carries with it an attractive and elucidating aspect, which, to many, is not present in the subtler methods of wave mechanics. For this reason, the constructor, the architect, the engineer, and all those accustomed to graphical analysis, should find this procedure of special interest.

Geometric acoustics, like geometric optics, is based on a number of postulates; the principal ones are:

1. Sound travels in straight lines in a homogeneous medium.
2. When two rays of sound intersect, the subsequent path of each is the same as though each ray existed separately.
3. When a ray is reflected, the angle of incidence equals the angle of reflection.

As in the case of optics, it is advisable to note carefully the limitations inherent in this method. They are, essentially, that a ray of sound is reflected geometrically only as long as the reflecting surface is large compared to the wave-length of the sound under consideration. When the dimensions of an obstacle are commensurate with, or smaller than, the wave-length, the sound will be diffracted,* that is, it will *bend* around the obstacle.

Single Reflections

Under this heading the following subjects will be considered:

1. Graphic representation of reflected sound
2. Ratio of direct to reflected sound
3. Interference effects
4. Reflection characteristics of surfaces

Graphic Representation of Reflected Sound

The simplest case of geometric acoustics is that of an isolated source of sound in front of a reflecting surface. This amounts essentially to a loud-speaker suspended at some distance above

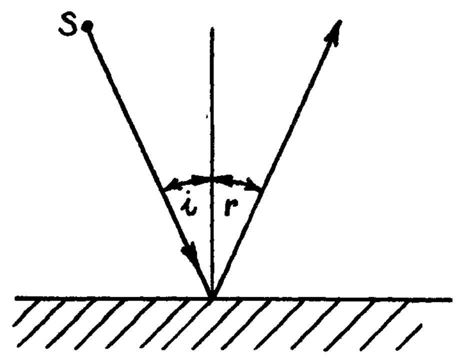


Figure 4

GRAPHICAL REPRESENTATION OF A SOUND RAY

the ground and radiating sound uniformly in all directions. Figure 4 shows the graphical representation of this condition. The angle i is known as the angle of incidence, and the angle r,

* Diffraction of sound should not be considered an *acoustic aberration*, occurring in defiance of geometric theory, but rather as a fundamental property of sound, of which the rectilinear propagation represents merely a special case. It is admittedly a difficult phenomenon to understand, and one which gives ground readily only to formal mathematical operation..

that of reflection. Each is measured between the respective ray and the normal to the surface. The source of sound is represented by S.

While Figure 4 pictures only one sound ray, Figure 5 shows a few of the infinite number of assumedly existent rays.

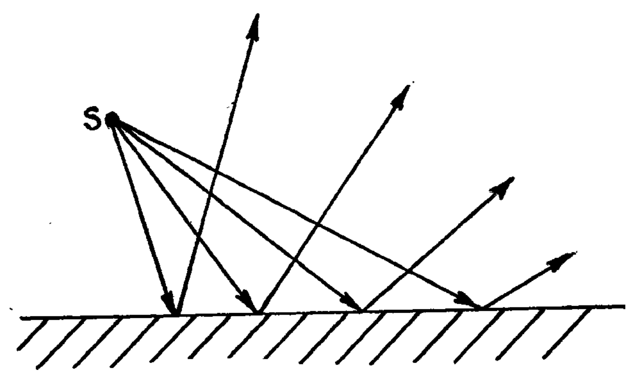


Figure 5

GRAPHICAL REPRESENTATION OF SEVERAL (OF THE ASSUMEDLY INFINITE NUMBER) OF THE SOUND RAYS FROM A SINGLE SOURCE

Figure 6 shows how the reflected ray can be constructed in a simple manner. *I* represents the *image* source of sound, which is located on a perpendicular to the surface, the perpendicular passing through the source S. The distance along the normal between image source and surface is the same as that between the real source and surface. For a direct sound ray incident on the surface, the direction of the corresponding reflected ray can be determined by drawing a line from the image through the point of incidence. While the path of a reflected ray gives information concerning only one point on the reflected wavefront, in practice the experienced operator or experimenter can frequently elicit much valuable information in this way.

Since all reflected sound may be considered to emanate from the image source, it becomes an easy matter to construct the entire reflected wavefront. This is done by drawing lines from

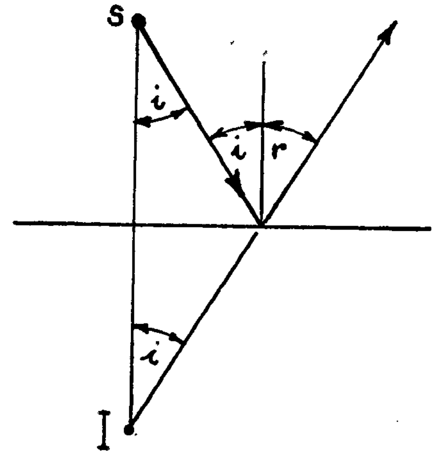


Figure 6

CONSTRUCTION OF A REFLECTED SOUND RAY

the image through the extreme points of the surface and by striking an arc between these rays with the center of the arc located at the image point. This is shown in Figure 7.

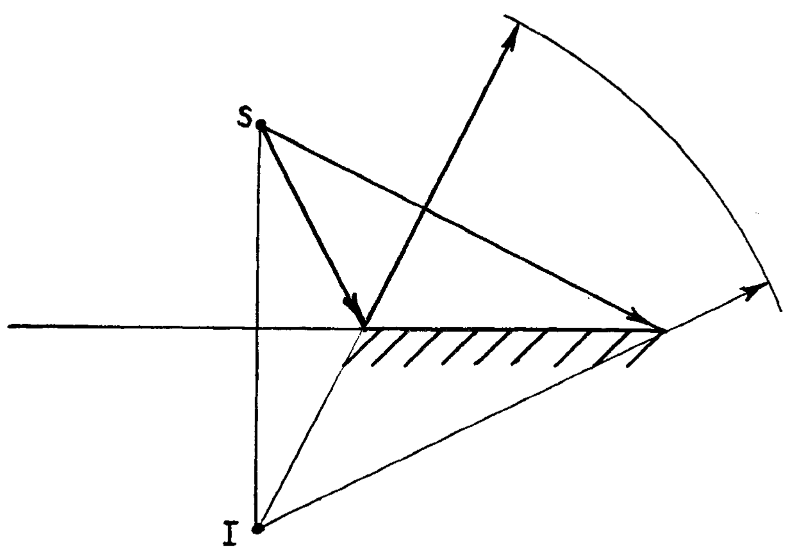


Figure 7

CONSTRUCTION OF A REFLECTED WAVEFRONT

Ratio of Direct to Reflected Sound

For the determination of the ratio of direct to reflected sound (without regard to their phase difference) at a point where the rays cross, it is desirable to consider several fundamental properties of sound. For a plane wave and a non-directional source, the sound pressure and particle velocity vary inversely with the distance from the source, while the sound energy varies inversely with the square of this distance. Similarly, the pressure reduction of sound striking an absorbent surface varies inversely as the square root of the energy reflectivity of the surface. The energy reduction, therefore, is inversely proportional to this reflectivity. For the simple case illustrated in Figure 8, write

$$\frac{P}{P'} = \frac{d}{r\sqrt{\beta}} = \frac{d}{r\sqrt{1-\alpha}}$$

$$\frac{E}{E'} = \left(\frac{d}{r}\right)^2 \frac{1}{\beta} = \left(\frac{d}{r}\right)^2 \frac{1}{(1-\alpha)}$$

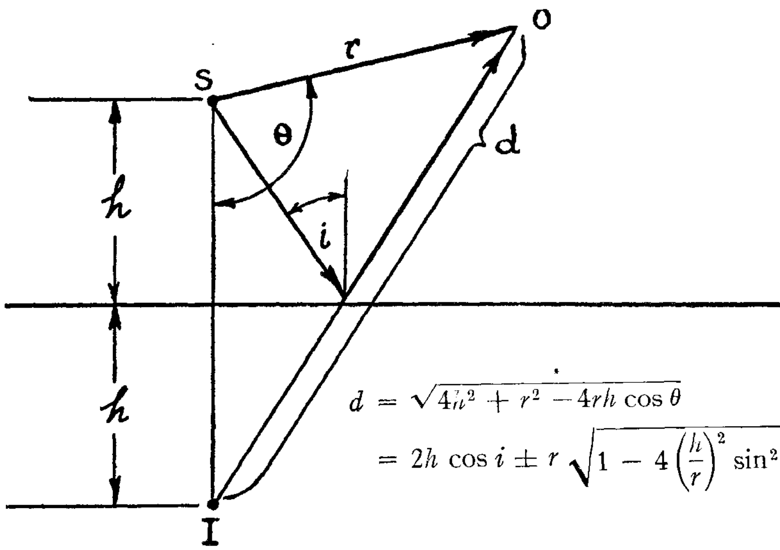
or

$$\left(\frac{P}{P'}\right)^2 = \frac{E}{E'} = \left(\frac{d}{r}\right)^2 \frac{1}{\beta} = \left(\frac{d}{r}\right)^2 \frac{1}{(1-\alpha)}$$

where P = pressure of direct sound
P' = pressure of reflected sound
E = energy density of direct sound
E' = energy density of reflected sound
r = distance travelled by direct sound
d = distance travelled by reflected sound
α = energy absorbtivity
β = energy reflectivity

The length of the reflected sound ray, in terms of the constants shown in Figure 8, is expressed by

$$d = \sqrt{4h^2 + r^2 - 4rh\cos\theta}$$
$$= 2h\cos i \pm r\sqrt{1 - 4\left(\frac{h}{r}\right)^2 \sin^2 i}$$



$$d = \sqrt{4h^2 + r^2 - 4rh\cos\theta}$$
$$= 2h\cos i \pm r\sqrt{1 - 4\left(\frac{h}{r}\right)^2 \sin^2 i}$$

Figure 8

THE LENGTH OF A REFLECTED SOUND RAY IN TERMS OF KNOWN CONSTANTS

If r is expressed in terms of h, e.g., $r = ah$,

$$d = h\sqrt{4 + a^2 - 4a\cos\theta}$$
$$= h\left[2\cos i \pm a\sqrt{1 - \left(\frac{2}{a}\right)^2 \sin^2 i}\right]$$

The ratio of direct to reflected sound pressure at the point O will be

$$Q = \frac{P}{P'} = \frac{d}{r\sqrt{1-\alpha}} = \frac{d}{ah\sqrt{1-\alpha}} = \frac{\sqrt{\left(\frac{2}{a}\right)^2 + 1 - \frac{4}{a}\cos\theta}}{\sqrt{1-\alpha}}$$
$$= \frac{\left[\frac{2}{a}\cos i \pm \sqrt{1 - \left(\frac{2}{a}\right)^2 \sin^2 i}\right]}{\sqrt{1-\alpha}}$$

Figure 9 shows in graphic form the distribution of the ratio Q, of direct to reflected sound pressure, for different ratios, a. For the sake of simplicity, the reflectivity of the surface is assumed to be 1 *. Thus, for instance, for the case where $r = h$, this ratio is 3 when θ is equal to 180 degrees. This means that at this point the direct sound pressure is three times as great as the pressure of the sound reflected from the surface.

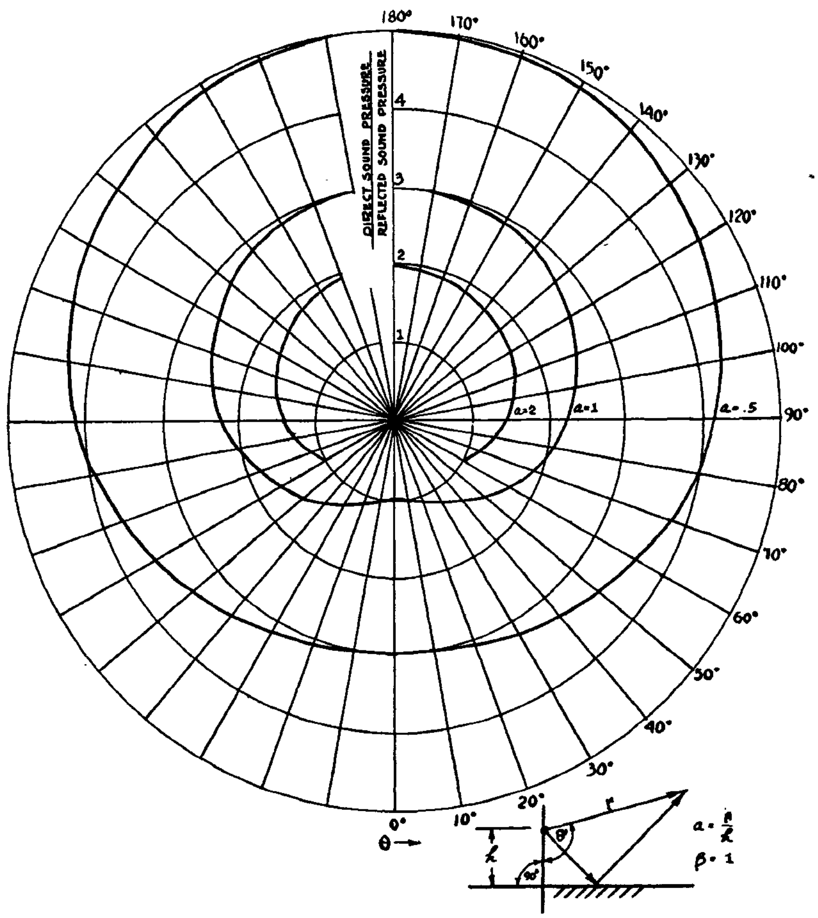


Figure 9

DISTRIBUTION OF THE RATIO Q, THE DIRECT TO REFLECTED SOUND PRESSURE, FOR THE CASE ILLUSTRATED IN THE FIGURE

* Frequently α and β are functions of θ. The case is idealized.

Figure 10 shows the variation of Q as a function of a, for different angles, θ.

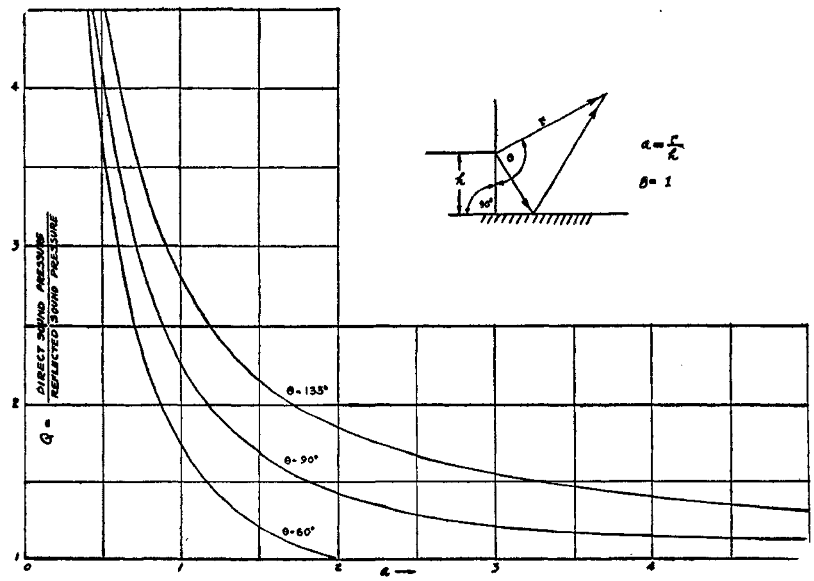


Figure 10

VARIATION OF Q, THE DIRECT TO REFLECTED SOUND PRESSURE, AS A FUNCTION OF THE RATIO, *a*, INDICATED IN THE FIGURE

Both Figures 9 and 10 are helpful in visualizing the sound pressure contributed by a single reflection from a highly reflective surface. Examples of this condition frequently occur in practice, as, in sound recording, when a person speaks near a wall, a window, or some other *hard* surface, which will reinforce his voice increasingly as he or the microphone approaches the reflecting surface.

To ascertain by how many decibels the direct sound pressure exceeds the reflected sound pressure, write

$$db = 20 \log Q = 10 \log \left[\left(\frac{2}{a} \right)^2 + 1 - \frac{4}{a} \cos \theta \right] - 10 \log \beta$$

Interference Effects

While the ratio Q as derived above holds true for all frequencies as long as the reflecting surface is large compared to the wave-length, the reflected sound will either reinforce or cancel part of the direct sound, depending on the phase relationship between the two rays at the point of observation. Only if the phase difference equals a wave-length, or one of its multiples, will complete reinforcement result. If the phase difference is $(2n-1)\,\lambda/2$, where λ is the wave-length and n any integer, the two sounds will be out of phase, and cancellation will take place, producing a so-called *dead spot.*

Mathematically, this may be stated as (see Figure 8)

$$\sqrt{4h^2 + r^2 - 4rh\cos\theta} - r = \frac{(2n-1)\lambda}{2}$$

or

$$\lambda = \frac{2[\sqrt{4h^2 + r^2 - 4rh\cos\theta} - r]}{2n-1}$$

The variation of wave-length (for which cancellation occurs) with distance of point of observation may be easily pictured quantitatively for the simplified case where θ is equal to 90 degrees. For this case, and writing $r = ah$, we obtain

$$\lambda = \frac{2h[\sqrt{4 + a^2} - a]}{2n-1}$$

This relationship is shown on Figure 11 for the case $n = 1$. The nearer the source, the longer the wave-length (or the lower the frequency) at which interference takes place. Similarly, the farther the point of observation moves from the source, the shorter the wave-length (or the higher the frequency) at which interference takes place. This is because the difference in length between the direct and reflected sound rays becomes progressively smaller, assuming of course that the distance of the source from the reflecting surface does not change and that the point of observation is moving along the line for which θ equals 90 degrees.

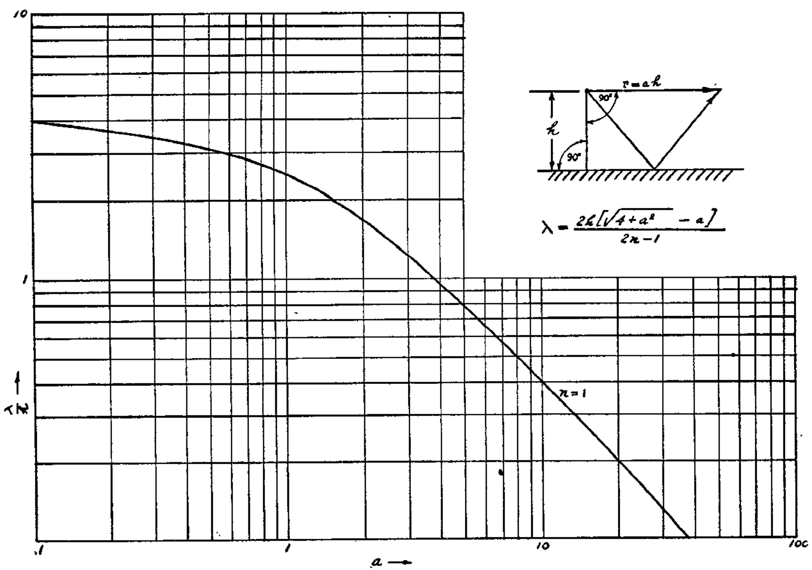


Figure 11

INTERFERENCE DIAGRAM FOR PLANE SURFACE

(The nearer the point of observation is moved to the source of sound, the lower the frequency at which interference can take place.)

It should not be inferred from the above, however, that at any one point in the sound field, interference can take place at only one frequency. Interference can take place at a number of frequencies, as is illustrated in Figure 12. The frequency at which an interference minimum occurs is expressed by

$$F = \frac{c(2n - 1)}{2[\sqrt{d^2 + 4h^2} - d]}$$

The frequency at which an interference maximum occurs is given by

$$F' = \frac{cn}{\sqrt{d^2 + 4h^2} - d}$$

where

$c =$ velocity of sound

$n = 1, 2, 3,$ etc.

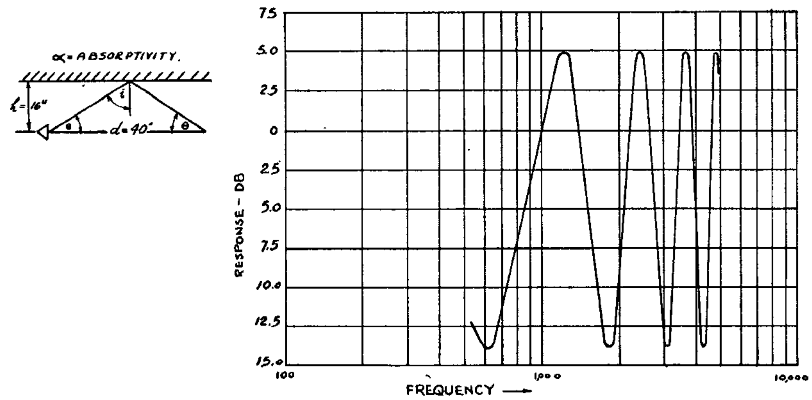


Figure 12

FREQUENCY CURVE

(At any one point of observation, interference can take place for a number of frequencies.)

Pertinent to Figure 12, reinforcement, expressed in decibels above the intensity of the direct sound at the point of observation, assuming both a non-directional source and sound pick-up device, is given by

$$DB = 20 \log_{10}\left[1 + \frac{d(1-\alpha)^{\frac{1}{2}}}{\sqrt{d^2 + 4h^2}}\right]$$
$$= 20 \log_{10}[1 + (1-\alpha)^{\frac{1}{2}} \cos\theta]$$
$$= 20 \log_{10}[1 + (1-\alpha)^{\frac{1}{2}} \sin i]$$

Similarly, the resultant sound intensity, expressed in decibels below direct-sound intensity, for the case of a 180-degree phase difference between direct and reflected sound ray, is given by

$$DB' = 20 \log_{10}\left[1 - \frac{d(1-\alpha)^{\frac{1}{2}}}{\sqrt{d^2 + 4h^2}}\right]$$
$$= 20 \log_{10}[1 - (1-\alpha)^{\frac{1}{2}} \cos\theta]$$
$$= 20 \log_{10}[1 - (1-\alpha)^{\frac{1}{2}} \sin i]$$

From the above equations it is seen that reinforcement at most can be only 6 decibels (when both θ and α equal zero), while the cancellation effect can assume an appreciable value, even close to the source and for moderately large values of α.

Reflection Characteristics of Surfaces

So far, only the effect of *sound rays* (a direct and a reflected ray) crossing each other has been considered. The ratio of their in-phase magnitudes has been established, and the conditions for cancellation and reinforcement between the two have been set down. The individual *amplitudes* of the sound rays, with their added magnitudes, or with the spatial distribution of the added amplitudes has not been discussed. For this purpose, some mathematical expressions describing the physical phenomena of sound will be used. The main interest is in the broader aspects of the problem, and it will be assumed that the observer is always far enough from the (non-directional) sound source to deal with plane rather than spherical waves. Therefore, sound pressures can be represented by simple sine waves, which can be added by the well-known laws of trigonometry.

The matter of interference produced by a *highly* reflecting surface may be satisfactorily examined by considering the image source as a duplicate source, with equal power output, free to radiate in all directions as though no separating wall existed. Consider Figure 13 and assume that a sine wave of unit amplitude is originating at both S and I. The resultant of the two waves, at a point far away from the sources, will be

$$R = \sin \omega t + \sin (\omega t + \varphi)$$

where

$$\omega = \frac{2\pi}{\lambda}$$

$$\lambda = \text{wave-length}$$

$$t = \text{time}$$

$$\varphi = \text{phase}$$

$$= \frac{2\pi g}{\lambda}$$

$$= \frac{2\pi d}{\lambda} \cos i$$

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