GRAPHICAL TECHNIQUES

for

ENGINEERING COMPUTATIONS

SCALES—TOOLS OF SCIENCE AND INDUSTRY

GRAPHICAL TECHNIQUES FOR ENGINEERING COMPUTATIONS

by

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To My Wife

CATHERINE

PREFACE

There is a steadily growing interest in the capabilities of graphical methods in the field of computation and an increasing demand for applications of these methods to a broad spectrum of scientific and engineering formulas—scientific principles or laws expressed in mathematical symbols.

Over the years, man's scientific endeavors have resulted in the accumulation of ponderous volumes of these formulas involving computation for their application to engineering problems. At the same time, man has developed a number of devices for reducing the labor of these computations—numerical devices such as the abacus and tables of logarithms, mechanical devices such as adding machines and desk calculators, electronic devices such as the modern computers, and graphical devices such as the slide rule and the nomograph. It is this last class of devices with which we are now concerned.

In a day that is witnessing the lightning invasion of high-speed desk calculators and electronic computers into the research laboratories, engineering departments, and business offices of industry, university, and government, one might well expect the comparatively simple graphical device—engineering workhorse of a decade or so ago—to dissolve gracefully into oblivion before the onslaught. Instead, the interest continues to grow.

The reason is quite simple. The calculation of a series of values required for the solution of an engineering problem (e.g., the design of a column for the fractionation of a hydrocarbon mixture) can be quickly performed with all required accuracy by the use of charts contained between the covers of a handbook at the engineer's fingertips. To use the electronic computer with equal ease and speed, it would be necessary that the engineer have at his fingertips both the computer and a program on punch cards or tape for the specific problem, and that he be able to enter his data without delay and receive his results immediately. Such, of course, is not the case. The computer is not at his fingertips, but in another room or possibly another building; the program for his problem, if it exists, is in a card file or tape repository, and must be fed separately to the machine; his data must be prepared in the proper form for entry; and his problem must await its turn in the schedule of a busy computer. Computer time and punch card programs are expensive in comparison with nomographs, many of which are available at little or no charge. For the computation of an extensive table of values based upon a single formula, the computer is economical, efficient, and remarkably fast, but in convenience and ease of operation it does not compete with the graphical device for many of the day-to-day computations of engineering. Indeed, by providing a high-speed method for processing and analyzing the mountains of data generated in today's unprecedented program of research, the computer is constantly spawning new formulas for which graphical methods offer the most satisfactory means of every-day application.

Many scientists and engineers that use graphical devices have little idea of the relative merits and applicability of the various types of devices, and virtually no knowledge of the underlying theory of their construction. Yet, the mathematics of this theory is so simple that mathematics advisers on projects for high school science fairs would do well to consider some of the methods described herein, such as construction of special slide rules, nomographs for formulas of current interest, and three-dimensional nomographs. There is no lack of technical literature in this field, but what is lacking is a systematic approach to the subject as a whole, from the standpoints of both organization and theory.

This book is an outgrowth of very earnest efforts towards unifying my own knowledge in this field. Having made a thorough study of nomographic methods and theory, I nevertheless found myself in poor shape to produce a series of nomographs based on certain polynomials describing the characteristics of flight of helicopters. A modification of existing theory greatly simplified the procedure for representing polynomials; however, it also pointed the way towards the development of a new theory of nomographic representation, the hyperbolic coordinate method from which have grown generalizations and extensions covering the entire field of nomography. At the same time, extension of the idea of the scale equation into the areas of graph papers, graphs, and slide rules has simplified the application of these devices in the field of computation.

Although some topics (e.g., the hyperbolic coordinate method of nomography) are treated in much detail because of the lack of thorough treatment elsewhere, other topics (e.g., graphical integration and differentiation) are recognized as being adequately covered in other sources and are here given only introductory discussions by way of recognizing their family relationships. Much of the material has been expanded from my original publications in this field; however, it will be obvious to the reader that I have also leaned heavily upon the works of such outstanding authors as d'Ocagne, Allcock and Jones, Lipka, and Mavis.

The illustrative examples and the problems that close each chapter represent many diverse disciplines, and they are derived largely from private correspondence with many that have discovered the value of graphical methods in their own fields; to these contributors I am pleased to express my appreciation. I am also deeply grateful to the many that contributed kindly advice and assistance as this work progressed, especially Mrs. Margaret Looper Weill and Mrs. Caren Burrows Yarbrough, who did some of the freehand sketches; Mr. Howard G. Dunlap and Mr. Edward Foster, who read portions of the manuscript; and the Engineering Experiment Station, Georgia Institute of Technology, which contributed generously to the support of the original research as well as to the preparation of the final manuscript.

November 1964 W. HERBERT BURROWS

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chapter 1

SCALES

All graphical methods of computation, indirectly or directly, are based upon scales. This fact makes it imperative that a clear and concise conception of the nature and properties of scales be acquired from the outset. To be sure, readers of this book have been acquainted with scales of one sort or another since infancy, and have used them to measure the dimensions of a room, to read the time of day, and to ascertain the temperature or the velocity of an automobile. But the background of experience thus acquired is of greater value in the application of graphical devices to the solution of problems than in the design of the instruments of computation themselves.

NATURE AND PROPERTIES OF SCALES

1.1 Some Familiar Scales

The familiar scales in the frontispiece display several features characteristic of all scales. Each is a line segment, whether straight or curved, with subdivisions marked by points or dashes along its length. The scale thus marked is said to be graduated; the points or dashes are called graduations. Each scale has a point (which may or may not appear on the segment shown) which is denoted as the origin of the scale. The positions of remaining points on the scale are determined by a functional relationship of the type,

$$s = mf(x) + l_o$$

in which s is the length of the line from the origin to the said point, m and l_o are constant, and f(x) is some function[1] of the variable x represented by the scale. This relationship is commonly known as the defining equation of the scale.

One scale shown is a foot-rule; divided into inches and sixteenths of inches, but showing the useful half-inch, quarter-inch, and eighth-inch subdivisions by dashes of different lengths. The defining equation of this scale is $s = x$, which is to say, the length of the segment of the scale is equal to the distance it measures. Here f(x) is x, itself. The value of m is unity, and that of l_o, zero. A later discussion of the meanings of m and l_o should make the reasons for these values clear.

Another scale shown is a thermometer scale, for which the defining equation is

$$s = mt + l_o$$

in which t is the temperature in degrees Fahrenheit, m is a proportionality constant, namely, the increase in length of the mercury thread for each degree rise in the temperature, and l_o is the length of the mercury thread when the temperature is 0°F.

Still another scale is a radio dial, graduated in kilocycles per second, a unit of frequency. Rotation of the dial in a clockwise direction decreases the capacitance of a condenser in the radio circuit, which, in turn, increases the frequency of the tuning circuit. Neither of these relationships is linear (i.e., in direct proportion), for the condenser is shaped in a manner such that equal angles of rotation do not produce equal changes of capacitance; furthermore, frequency is inversely, not directly, proportional to capacitance of the condenser. The nature of f( ) is determined by this inverse relationship and by the shape of the condenser blades, which differs in various makes. Actually, the positions of the dial corresponding to the frequencies shown were probably ob-

1 By "function of x" is meant a quantity whose value is fixed for each fixed value of x. In a pure sense, f(x) may be a constant; i.e., it may have the same value for all values of x. In all of the present cases, f(x) is a variable, taking on different values for the different values of x.

tained by calibrating an identical condenser against standard frequencies. The exact nature of $f(\nu)$ is of no importance, and the scale may be quite satisfactorily defined by the relationship

$$s = mf(\nu) + l_0$$

The frequency is represented by ν and m is a proportionality constant, the increase in length of scale for each increase of unity in the value of $f(\nu)$. Because of the inverse relationship of $f(x)$ to x, l_0 is an infinite length, the length of the scale when $f(x)$ is zero.

1.2 Modulus

In the defining equation of the temperature scale, the term m is identified as the increase in length of the mercury thread corresponding to an increase of 1°F in the temperature, for in this case the "function of the temperature" is the temperature itself. In the defining equation of the scale of a radio dial, the term m is not a length of scale corresponding to an increase of unity in the frequency, for this length varies over the entire length of the scale. It is, instead, the length of scale corresponding to an increase of unity in the value of the "function of the frequency." This term, which is the length of scale corresponding to an increase of unity in the value of the function represented, is known as the modulus of the scale.

The scale is constructed by plotting the positions of its graduations on a reference scale, or reference frame. The reference frame may be an axis of the cartesian coordinate system—another scale, or simply a straight line having on it two points with assigned values. The point on the reference frame having the assigned value of zero is known as the origin. The origin of the reference frame does not necessarily correspond to the zero point of the scale, which is the position of the point $f(x) = 0$ on the reference frame. Figure 1:1 shows three logarithmic scales all having the same modulus (2.5 inches) but differing in the position of its zero point on the reference frame. The zero point of scale A coincides with the origin of the reference frame. The zero point of scale B lies 1.5 inches to the left of the origin; that of scale C lies four inches to the right of the origin of the reference frame. These positions are defined by the values of l_o in the defining equations, which are, respectively, $s = 2.5 \log x$, $s = 2.5 \log x - 2.5$, and $s = 2.5 \log x + 4$.

1.3 Some Useful Types of Scales

Some of the more useful types of scales are shown in Figure 1:2. These scales are extensively used in the con-

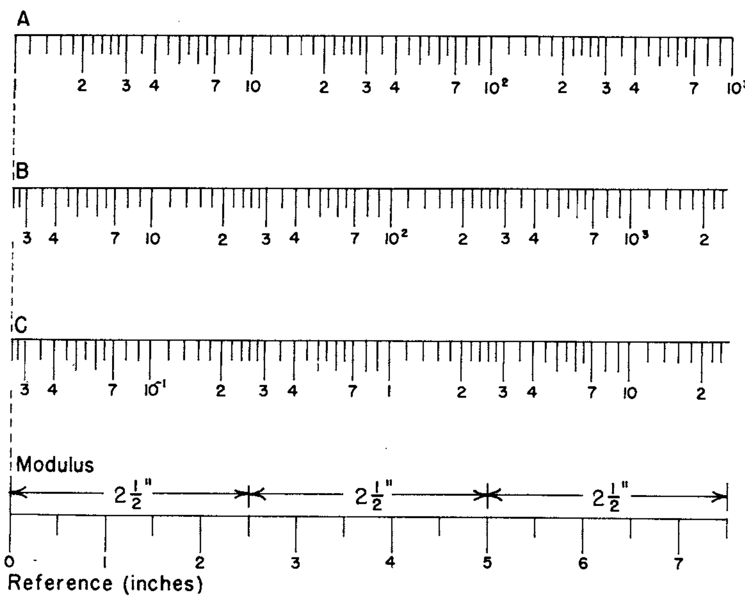

FIG. 1:1 LOGARITHMIC SCALES, MODULUS 2.5 INCHES, WITH DIFFERENT ZERO POINTS
A) $s = 2.5 \log x$; B) $s = 2.5 \log x - 1.5$;
C) $s = 2.5 \log x + 4.0$

struction of graph papers, nomographs, and slide rules. The characteristic defining equations of these scales are

$$\text{Linear:} \qquad x = mx + l_o$$

$$\text{Logarithmic:} \qquad s = m \log x + l_o$$

$$\text{Reciprocal:} \quad s = m/x + l_o$$

$$\text{Hyperbolic:} \quad s = \frac{mx}{x + r} + l_o$$

The last of these is shown for the case $r = 1$.

1.4 Errors in Absolute and Relative Scales

It must be recognized from the beginning that accuracy in the use of a scale depends upon one's locating the exact point

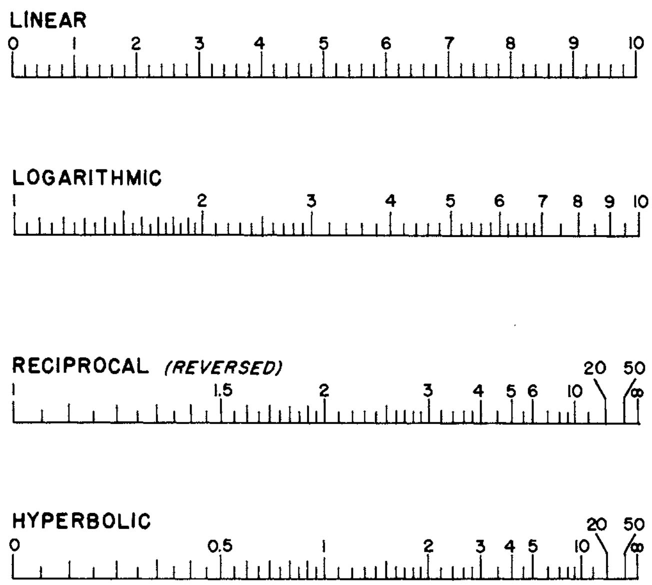

FIG. 1:2 TYPICAL LINEAR, LOGARITHMIC, RECIPROCAL, HYPERBOLIC SCALES

representing the desired value of the variable that the scale represents. As this is virtually impossible, there is always an error, however slight, represented by the difference in the position of the point actually located and the position of the desired point. If the value of x at the desired point is x_1, and that at the point actually located is x_2, then the difference $x_2 - x_1$ is the error in reading the value of x at that point.

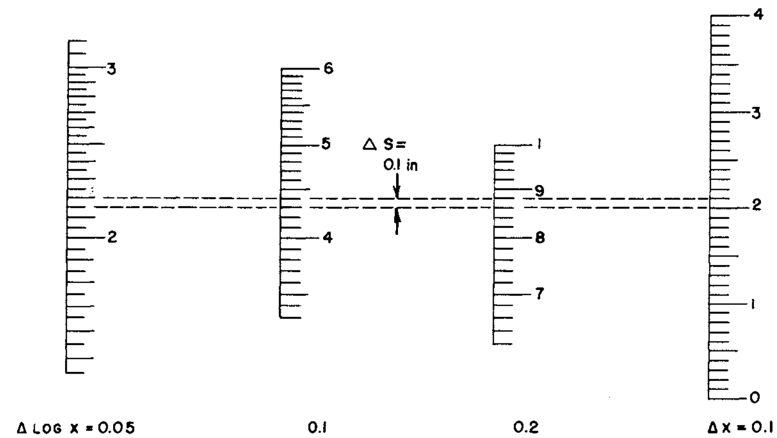

FIG. 1:3 ABSOLUTE AND RELATIVE ERRORS OF LINEAR AND LOGARITHMIC SCALES

This difference may be designated Δx; then $\Delta x = x_2 - x_1$. The absolute error of the scale is the ratio of the error in the value of the reading to the error in position of the point, or $\Delta x/\Delta s$. To confine attention to a single point, it is necessary that the increments Δx and Δs be infinitesimal, whereupon the value of the absolute error becomes

$$\text{Absolute error} = \frac{dx}{ds}$$

In essence, this factor dx/ds determines the magnitude of the error which will result from a given error in locating a scale point.

In financial matters, virtually all computation is performed with the same absolute error; regardless of the size of the budget, it must be accounted to the penny. This is not the case in numerous other fields, wherein errors of measurement are related to the size of the measurement. In such cases, the relative error has greater significance. The relative error is defined as the ratio of the value of the absolute error to the value of the variable being measured, or

$$\text{Relative Error} = \frac{dx/ds}{x}$$

Absolute and relative errors of the linear and logarithmic scales at various positions are shown in Figure 1:3. Δs is taken as 1/10 inch, which is large compared with the infinitesimal, ds; consequently, the estimates given for the values of the errors on the logarithmic scale cannot be exactly equal to the true values obtained from the formulas. They are close enough, however, to show the significance of these errors. It may be noted that the linear scale has the same absolute error throughout its length. That is, the absolute error of the linear scale is independent of the variable represented. This

TABLE 1:1

ERRORS IN SCALES

Type	Defining Equation	Absolute Error	Relative Error
Linear	$s = mx + l_o$	$\frac{1}{m}$	$\frac{1}{mx}$
Logarithmic	$s = m \log x + l_o$	$\frac{x}{m}$	$\frac{1}{m}$
Reciprocal	$s = \frac{m}{x} + l_o$	$\frac{-x^2}{m}$	$\frac{-x}{m}$
Hyperbolic	$s = \frac{mx}{x + r} + l_o$	$\frac{(x + r)^2}{rm}$	$\frac{(x + r)^2}{rmx}$

is not true on the logarithmic scale, where the absolute error increases with the value of the variable; the logarithmic scale has the same relative error throughout its length.

The absolute and relative errors of the scales shown in Figure 1:2 are given in Table 1:1.

1.5 Non-linear Reference Frames

The usual reference frame for the construction of scales for graphical computations is an axis of the cartesian coordinate system; these coordinates are linear. It is occasionally necessary, however, to construct a scale upon some other base, as, for example, an hyperbolic coordinate axis. In these cases, the scales are designated as such by name, using the name of the function followed by the name of the reference

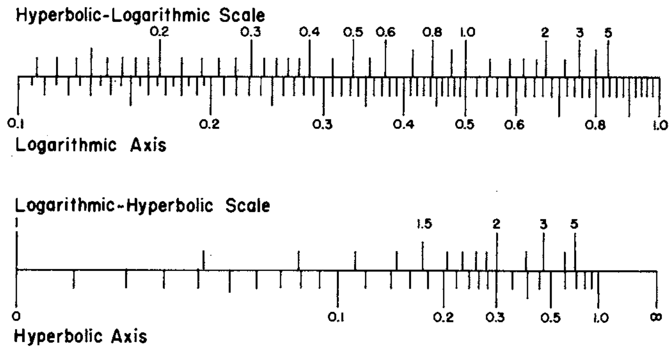

FIG. 1:4 LOGARITHMIC-HYPERBOLIC AND HYPERBOLIC-LOGARITHMIC SCALES

frame. For instance, a logarithmic-hyperbolic scale is a scale whose graduations are computed from a logarithmic function and plotted on an hyperbolic reference frame. Figure 1:4 shows a logarithmic-hyperbolic scale and an hyperbolic-logarithmic scale. Obviously, the two are not identical.

METHODS FOR CONSTRUCTING SCALES

1.6 Construction of a Scale from Data

If the value of s for each value of x that is to be plotted has been computed, or exists in the form of a data table, the scale may be constructed by locating each scale point on the reference frame. Table 1:2 shows the vapor pressures of water over a range of 0° to 100°C, which may be used as a basis for the construction of a scale of vapor pressures as functions of temperature.

As v.p. = f(t), the defining equation of the scale is

$$s_t = m(\text{v.p.})_t - l_o$$

l_o may be taken as zero; the modulus m is selected to accommodate the range desired within the length of the scale:

$$\text{modulus} = \frac{\text{length}}{\text{range of function}}$$

TABLE 1:2

VAPOR PRESSURES OF WATER
(in mm of Mercury)

t, °C	V.P., mm	t, °C	V.P., mm	t, °C	V.P., mm	t, °C	V.P., mm
0	4.58	58	136.08	73	265.7	87	468.7
10	9.21	60	149.38	74	277.2	88	487.1
20	17.54	61	156.43	75	289.1	89	506.1
25	23.76	62	163.77	76	301.4	90	525.8
30	31.82	63	171.38	77	314.1	91	546.1
35	42.18	64	179.31	78	327.3	92	567.0
40	55.32	65	187.54	79	341.0	93	588.6
42	61.50	66	196.09	80	355.1	94	610.9
44	68.26	67	204.96	81	369.7	95	633.9
46	75.65	68	214.17	82	384.9	96	657.6
48	83.71	69	223.73	83	400.6	97	682.1
50	92.51	70	233.7	84	416.8	98	707.3
52	102.09	71	243.9	85	433.6	99	733.2
54	112.51	72	254.6	86	450.9	100	760.0
56	123.80						

In this case the range of v.p. is approximately 800 mm. and the length of scale desired is eight inches. The modulus is then 8 inches/800 mm. = 0.01 inch/mm. It follows that 1 inch = 100 mm. (Fig. 1:5).

The reference frame is a scale divided into inches and tenths of inches, marked with the values of vapor pressure. Points corresponding to the values of vapor pressure for each value of temperature are marked off on this reference frame, and the temperatures to which they correspond are printed on the scale.

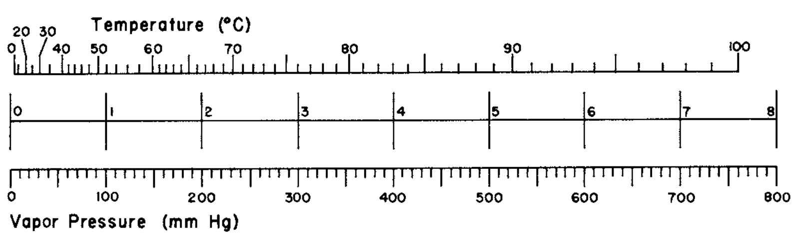

FIG. 1:5 SCALE SHOWING VAPOR PRESSURES OF WATER AS A FUNCTION OF TEMPERATURE, °C

APPENDIX

TABLE

Values of $x = \frac{p}{p + r}$ and $1 - x$

p/r	x	1 - x	p/r	x	1 - x
.000	.00000	1.00000	.046	.04398	.95602
.001	.00100	.99900	.047	.04489	.95511
.002	.00200	.99800	.048	.04580	.95420
.003	.00299	.99701	.049	.04671	.95329
.004	.00398	.99602	.050	.04762	.95238
.005	.00498	.99502	.051	.04853	.95147
.006	.00596	.99404	.052	.04943	.95057
.007	.00695	.99305	.053	.05033	.94967
.008	.00794	.99206	.054	.05123	.94877
.009	.00892	.99108	.055	.05213	.94787
.010	.00990	.99010	.056	.05303	.94697
.011	.01088	.98912	.057	.05393	.94607
.012	.01186	.98814	.058	.05482	.94518
.013	.01283	.98717	.059	.05571	.94429
.014	.01381	.98619	.060	.05660	.94340
.015	.01478	.98522	.061	.05749	.94251
.016	.01575	.98425	.062	.05838	.94162
.017	.01672	.98328	.063	.05927	.94073
.018	.01768	.98232	.064	.06015	.93985
.019	.01865	.98135	.065	.06103	.93897
.020	.01961	.98039	.066	.06191	.93809
.021	.02057	.97943	.067	.06279	.93721
.022	.02153	.97847	.068	.06367	.93633
.023	.02248	.97752	.069	.06455	.93545
.024	.02344	.97656	.070	.06542	.93458
.025	.02439	.97561	.071	.06629	.93371
.026	.02534	.97466	.072	.06716	.93284
.027	.02629	.97371	.073	.06803	.93197
.028	.02724	.97276	.074	.06890	.93110
.029	.02818	.97182	.075	.06977	.93023
.030	.02913	.97087	.076	.07063	.92937
.031	.03007	.96993	.077	.07150	.92850
.032	.03101	.96899	.078	.07236	.92764
.033	.03195	.96805	.079	.07322	.92678
.034	.03288	.96712	.080	.07407	.92593
.035	.03382	.96618	.081	.07493	.92507
.036	.03475	.96525	.082	.07579	.92421
.037	.03568	.96432	.083	.07664	.92336
.038	.03661	.96139	.084	.07749	.92251
.039	.03754	.96246	.085	.07834	.92161
.040	.03846	.96152	.086	.07919	.92081
.041	.03939	.96061	.087	.08004	.91996
.042	.04031	.95969	.088	.08088	.91912
.043	.04123	.95877	.089	.08173	.91827
.044	.04215	.95785	.090	.08257	.91743
.045	.0430[illegible]	.95694	.091	.08341	.91659

p/r	x	x - x	p/r	x	1 - x
.092	.08425	.91575	.143	.12511	.87489
.093	.08509	.91491	.144	.12587	.87413
.094	.08592	.91408	.145	.12664	.87336
.095	.08676	.91324	.146	.12740	.87260
.096	.08759	.91241	.147	.12816	.87184
.097	.08842	.91158	.148	.12892	.87108
.098	.08925	.91075	.149	.12968	.87032
.099	.09008	.90992	.150	.13044	.86956
.100	.09091	.90909	.151	.13119	.86881
.101	.09174	.90826	.152	.13194	.86806
.102	.09256	.90744	.153	.13270	.86730
.103	.09338	.90662	.154	.13345	.86655
.104	.09420	.90580	.155	.13420	.86580
.105	.09502	.90498	.156	.13495	.86505
.106	.09584	.90416	.157	.13570	.86430
.107	.09666	.90334	.158	.13644	.86356
.108	.09747	.90253	.159	.13719	.86281
.109	.09829	.90171	.160	.13793	.86207
.110	.09910	.90090	.161	.13876	.86133
.111	.09991	.90009	.162	.13942	.86058
.112	.10072	.89928	.163	.14016	.85984
.113	.10153	.89847	.164	.14089	.85911
.114	.10233	.89767	.165	.14163	.85837
.115	.10314	.89686	.166	.14237	.85763
.116	.10394	.89606	.167	.14310	.85690
.117	.10475	.89525	.168	.14384	.85616
.118	.10555	.89445	.169	.14457	.85543
.119	.10635	.89365	.170	.14530	.85470
.120	.10714	.89286	.171	.14603	.85397
.121	.10794	.89206	.172	.14676	.85324
.122	.10873	.89127	.173	.14749	.85251
.123	.10953	.89047	.174	.14821	.85179
.124	.11032	.88968	.175	.14894	.85106
.125	.11111	.88889	.176	.14966	.85034
.126	.11190	.88810	.177	.15038	.84962
.127	.11269	.88731	.178	.15110	.84890
.128	.11348	.88652	.179	.15182	.84818
.129	.11426	.88574	.180	.15254	.84746
.130	.11504	.88496	.181	.15326	.84674
.131	.11583	.88417	.182	.15398	.84602
.132	.11661	.88339	.183	.15469	.84531
.133	.11739	.88261	.184	.15541	.84459
.134	.11817	.88183	.185	.15612	.84388
.135	.11894	.88106	.186	.15683	.84317
.136	.11972	.88028	.187	.15754	.84246
.137	.12049	.87951	.188	.15825	.84175
.138	.12127	.87873	.189	.15896	.84104
.139	.12204	.87796	.190	.15966	.84034
.140	.12281	.87719	.191	.16037	.83963
.141	.12358	.87642	.192	.16107	.83893
.142	.12434	.87566	.193	.16178	.83822

p/r	x	1 - x	p/r	x	1 - x
.194	.16248	.83752	.245	.19679	.80321
.195	.16318	.83682	.246	.19743	.80257
.196	.16388	.83612	.247	.19808	.80192
.197	.16458	.83542	.248	.19872	.80128
.198	.16528	.83472	.249	.19936	.80064
.199	.16597	.83403	.250	.20000	.80000
.200	.16667	.83333	.251	.20064	.79936
.201	.16736	.83264	.252	.20128	.79872
.202	.16805	.83195	.253	.20192	.79808
.203	.16875	.83125	.254	.20255	.79745
.204	.16944	.83056	.255	.20319	.79681
.205	.17012	.82988	.256	.20382	.79618
.206	.17081	.82919	.257	.20446	.79554
.207	.17150	.82850	.258	.20509	.79491
.208	.17219	.82781	.259	.20572	.79428
.209	.17287	.82713	.260	.20635	.79365
.210	.17355	.82645	.261	.20698	.79302
.211	.17424	.82576	.262	.20761	.79239
.212	.17492	.82508	.263	.20823	.79177
.213	.17560	.82440	.264	.20886	.79114
.214	17628	.82372	.265	.20949	.79051
.215	.17696	.82304	.266	.21011	.78989
.216	.17763	.82237	.267	.21073	.78927
.217	.17831	.82169	.268	.21136	.78864
.218	.17898	.82102	.269	.21198	.78802
.219	.17966	.82034	.270	.21260	.78740
.220	.18033	.81967	.271	.21322	.78678
.221	.18100	.81900	.272	.21384	.78616
.222	.18167	.81833	.273	.21445	.78555
.223	.18234	.81766	.274	.21507	.78493
.224	.18301	.81699	.275	.21569	.78431
.225	.18367	.81633	.276	.21630	.78370
.226	.18434	.81566	.277	.21692	.78308
227	.18500	.81500	.278	.21753	.78247
.228	.18567	.81433	.279	.21814	.78186
.229	.18633	.81367	.280	.21875	.78125
.230	.18699	.81301	.281	.21936	.78064
.231	.18765	.81235	.282	.21997	.78003
.232	.18831	.81169	.283	.22058	.77942
.233	.18897	.81103	.284	.22118	.77882
.234	.18963	.81037	.285	.22179	.77821
.235	.19028	.80972	.286	.22240	.77760
.236	.19094	.80906	.287	.22300	.77700
.237	.19159	.80841	.288	.22360	.77640
.238	.19225	.80775	.289	.22421	.77579
.239	.19290	.80710	.290	.22481	.77519
.240	.19355	.80645	.291	.22541	.77459
.241	.19420	.80580	.292	.22601	.77399
.242	.19485	.80515	.293	.22661	.77339
.243	.19550	.80450	.294	.22720	.77280
.244	.19614	.80386	.295	.22780	.77220

p/r	x	1 - x	p/r	x	1 - x
.296	.22840	.77160	.396	.28367	.71633
.297	.22900	.77100	.398	.28469	.71531
.298	.22958	.77042	.400	.28571	.71429
.299	.23018	.76982	.402	.28673	.71327
.300	.23077	.76923	.404	.28775	.71226
.302	.23195	.76805	.406	.28876	.71124
.304	.23313	.76687	.408	.28977	.71023
.306	.23430	.76570	.410	.29078	.70922
.308	.23547	.76453	.412	.29178	.70822
.310	.23664	.76336	.414	.29279	.70721
.312	.23781	.76219	.416	.29379	.70621
.314	.23897	.76103	.418	.29478	.70522
.316	.24012	.75988	.420	.29577	.70423
.318	.24127	.75873	.422	.29677	.70323
.320	.24242	.75758	.424	.29775	.70225
.322	.24357	.75643	.426	.29874	.70126
.324	.24471	.75529	.428	.29972	.70028
.326	.24585	.75415	.430	.30070	.69930
.328	.24699	.75301	.432	.30168	.69832
.330	.24812	.75188	.434	.30265	.69735
.332	.24925	.75075	.436	.30362	.69638
.334	.25038	.74962	.438	.30459	.69541
.336	.25150	.74850	.440	.30556	.69444
.338	.25262	.74738	.442	.30652	.69348
.340	.25373	.74627	.444	.30748	.69252
.342	.25484	.74516	.446	.30844	.69156
.344	.25595	.74405	.448	.30939	.69061
.346	.25706	.74294	.450	.31034	.68966
.348	.25816	.74184	.452	.31129	.68871
.350	.25926	.74074	.454	.31224	.68776
.352	.26036	.73964	.456	.31319	.68681
.354	.26145	.73855	.458	.31413	.68587
.356	.26254	.73746	.460	.31507	.68493
.358	.26362	.73638	.462	.31601	.68399
.360	.26471	.73529	.464	.31694	.68306
.362	.26579	.73421	.466	.31787	.68213
.364	.26686	.73314	.468	.31880	.68120
.366	.26794	.73206	.470	.31973	.68027
.368	.26901	.73099	.472	.32068	.67935
.370	.27007	.72993	.474	.32157	.67843
.372	.27114	.72886	.476	.32249	.67751
.374	.27220	.72780	.478	.32341	.67659
.376	.27326	.72674	.480	.32432	.67568
.378	.27431	.72569	.482	.32524	.67476
.380	.27536	.72464	.484	.32615	.67385
.382	.27641	.72359	.486	.32705	.67295
.384	.27746	.72254	.488	.32796	.67204
.386	.27850	.72150	.490	.32886	.67114
.388	.27954	.72046	.492	.32976	.67024
.390	.28058	.71942	.494	.33066	.66934
.392	.28161	.71839	.496	.33155	.66845
.394	.28264	.71736	.498	.33244	.66756

p/r	x	1 - x	p/r	x	1 - x
.500	.33333	.66667	.602	.37578	.62422
.502	.33422	.66578	.604	.37656	.62344
.504	.33511	.66489	.606	.37733	.62267
.506	.33599	.66401	.608	.37811	.62189
.508	.33687	.66313	.610	.37888	.62112
.510	.33775	.66225	.612	.37965	.62035
.512	.33862	.66138	.614	.38042	.61958
.514	.33950	.66050	.616	.38119	.61881
.516	.34037	.65963	.618	.38195	.61805
.518	.34124	.65876	.620	.38272	.61728
.520	.34211	.65789	.622	.38347	.61652
.522	.34297	.65703	.624	.38424	.61576
.524	.34383	.65617	.626	.38499	.61501
.526	.34469	.65531	.628	.38575	.61425
.528	.34555	.65445	.630	.38650	.61350
.530	.34641	.65359	.632	.38725	.61275
.532	.34726	.65274	.634	.38800	.61200
.534	.34811	.65189	.636	.38875	.61125
.536	.34896	.65104	.638	.38950	.61050
.538	.34980	.65020	.640	.39024	.60976
.540	.35065	.64935	.642	.39099	.60901
.542	.35149	.64851	.644	.39173	.60827
.544	.35233	.64767	.646	.39247	.60753
.546	.35317	.64683	.648	.39320	.60680
.548	.35401	.64599	.650	.39394	.60606
.550	.35484	.64516	.652	.39467	.60533
.552	.35567	.64433	.654	.39541	.60459
.554	.35650	.64350	.656	.39614	.60386
.556	.35733	.64267	.658	.39686	.60314
.558	.35815	.64185	.660	.39759	.60241
.560	.35897	.64103	.662	.39832	.60168
.562	.35980	.64020	.664	.39904	.80096
.564	.36061	.63939	.666	.39976	.60024
.566	.36143	.63857	.668	.40048	.59952
.568	.36224	.63776	.670	.40120	.59880
.570	.36306	.63694	.672	.40191	.59809
.572	.36387	.63613	.674	.40263	.59737
.574	36468	.63532	.676	.40334	59666
.576	.36548	.63452	.678	.40405	.59595
.578	.36629	.63371	.680	.40476	.59524
.580	.36709	.63291	.682	.40547	.59453
.582	.36789	.63211	.684	.40618	.59382
.584	.36869	.63131	.686	.40688	.59312
.586	.36948	.63052	.688	.40758	.59242
.588	.37028	.62972	.690	.40828	.59172
.590	.37107	.62893	.692	.40898	.59102
.592	.37186	.62814	.694	.40968	.59033
.594	.37265	.62735	.696	.41038	.58962
.596	.37343	.62657	.698	.41107	.58893
.598	.37422	.62578	.700	.41176	.58824
.600	.37500	.62500	.702	.41246	.58754

p/r	x	1 - x	p/r	x	1 - x
.704	.41315	.58685	.806	.44629	.55371
.706	.41383	.58617	.808	.44690	.55310
.708	.41452	.58548	.810	.44751	.55249
.710	.41520	.58480	.812	.44812	.55188
.712	.41589	.58411	.814	.44873	.55127
.714	.41657	.58343	.816	.44934	.55066
.716	.41725	.58275	.818	.44994	.55006
.718	.41793	.58207	.820	.45055	.54945
.720	.41860	.58140	.822	.45115	.54885
.722	.41928	.58072	.824	.45175	.54825
.724	.41995	.58005	.826	.45235	.54765
.726	.42063	.57937	.828	.45295	.54705
.728	.42130	.57870	.830	.45355	.54645
.730	.42197	.57803	.832	.45415	.54585
.732	.42263	.57737	.834	.45474	.54526
.734	.42330	.57670	.836	.45534	.54466
.736	.42396	.57604	.838	.45593	.54407
.738	.42463	.57537	.840	.45652	.54348
.740	.42529	.57471	.842	.45711	.54289
.742	.42595	.57405	.844	.45770	.54230
.744	.42661	.57339	.846	.45829	.54171
.746	.42726	.57274	.848	.45887	.54113
.748	.42792	.57208	.850	.45946	.54054
.750	.42857	.57143	.852	.46004	.53996
.752	.42922	.57078	.854	.46063	.53937
.754	.42987	.57013	.856	.46121	.53879
.756	.43052	.56948	.858	.46179	.53821
.758	.43117	.56883	.860	.46237	.53763
.760	.43182	.56818	.862	.46294	.53706
.762	.43246	.56754	.864	.46352	.53648
.764	.43311	.56689	.866	.46409	.53591
.766	.43375	.56625	.868	.46467	.53533
.768	.43439	.56561	.870	.46524	.53476
.770	.43503	.56497	.872	.46581	.53419
.772	.43567	.56433	.874	.46638	.53362
.774	.43630	.56370	.876	.46695	.53305
.776	.43696	.56304	.878	.46752	.53248
.778	.43757	.56243	.880	.46809	.53191
.780	.43820	.56180	.882	.46865	.53135
.782	.43883	.56117	.884	.46921	.53079
.784	.43946	.56054	.886	.46978	.53022
.786	.44009	.55991	.888	.47034	.52966
.788	.44072	.55928	.890	.47090	.52910
.790	.44134	.55866	.892	.47146	.52854
.792	.44196	.55804	.894	.47202	.52798
.794	.44259	.55741	.896	.47257	.52743
.796	.44321	.55679	.898	.47313	.52687
.798	.44383	.55617	.900	.47368	.52632
.800	.44444	.55556	.902	.47424	.52576
.802	.44506	.55494	.904	.47479	.52521
.804	.44568	.55432	.906	.47534	.52466

p/r	x	1 - x	p/r	x	1 - x
.908	.47589	.52411	1.025	.50617	.49383
.910	.47644	.52356	1.030	.50739	.49261
.912	.47699	.52301	1.035	.50860	.49140
.914	.47753	.52247	1.040	.50980	.49020
.916	.47808	.52192	1.045	.51100	.48900
.918	.47862	.52138	1.050	.51220	.48780
.920	.47917	.52083	1.055	.51338	.48662
.922	.47971	.52029	1.060	.51456	.48544
.924	.48025	.51975	1.065	.51574	.48426
.926	.48079	.51921	1.070	.51691	.48309
.928	.48133	.51867	1.075	.51807	.48193
.930	.48187	.51813	1.080	.51923	.48077
.932	.48240	.51760	1.085	.52038	.47962
.934	.48294	.51706	1.090	.52153	.47847
.936	.48347	.51653	1.095	.52267	.47733
.938	.48400	.51600	1.100	.52381	.47619
.940	.48454	.51546	1.105	.52494	.47505
.942	.48507	.51493	1.110	.52607	.47393
.944	.48560	.51440	1.115	.52719	.47281
.946	.48613	.51387	1.120	.52830	.47170
.948	.48665	.51335	1.125	.52941	.47059
.950	.48718	.51282	1.130	.53052	.46948
.952	.48770	.51230	1.135	.53162	.46638
.954	.48823	.51177	1.140	.53271	.46729
.956	.48875	.51125	1.145	.53380	.46620
.958	.48927	.51073	1.150	.53488	.46512
.960	.48980	.51020	1.155	.53596	.46404
.962	.49032	.50968	1.160	.53704	.46296
.964	.49084	.50917	1.165	.53811	.46189
.966	.49135	.50865	1.170	.53917	.46083
.968	.49187	.50813	1.175	.54023	.45977
.970	.49239	.50761	1.180	.54128	.45872
.972	.49290	.50710	1.185	.54233	.45767
.974	.49341	.50659	1.190	.54338	.45662
.976	.49393	.50607	1.195	.54442	.45558
.978	.49444	.50556	1.200	.54545	.45455
.980	.49495	.50505	1.205	.54649	.45351
.982	.49546	.50454	1.210	.54751	.45249
.984	.49597	.50403	1.215	.54853	.45147
.986	.49648	.50352	1.220	.54955	.45045
.988	.49698	.50302	1.225	.55056	.44944
.990	.49749	.50251	1.230	.55157	.44843
.992	.49799	.50201	1.235	.55257	.44743
.994	.49850	.50150	1.240	.55357	.44643
.996	.49900	.50100	1.245	.55457	.44543
.998	.49950	.50050	1.250	.55556	.44444
1.000	.50000	.50000	1.255	.55654	.44346
1.005	.50125	.49875	1.260	.55752	.44248
1.010	.50249	.49751	1.265	.55850	.44150
1.015	.50372	.49628	1.270	.55947	.44053
1.020	.50495	.49505	1.275	.56044	.43956

p/r	x	1 - x	p/r	x	1 - x
1.280	.56140	.43860	1.535	.60552	.39448
1.285	.56236	.43764	1.540	.60630	.39370
1.290	.56332	.43668	1.545	.60707	.39293
1.295	.56427	.43573	1.550	.60784	.39216
1.300	.56522	.43478	1.555	.60861	.39139
1.305	.56616	.43384	1.560	.60937	.39063
1.310	.56710	.43290	1.565	.61014	.38986
1.315	.56803	.43197	1.570	.61089	.38911
1.320	.56897	.43103	1.575	.61165	.38835
1.325	.56989	.43011	1.580	.61240	.38760
1.330	.57081	.42919	1.585	.61315	.38685
1.335	.57173	.42827	1.590	.61390	.38610
1.340	.57265	.42735	1.595	.61464	.38536
1.345	.57356	.42644	1.600	.61538	.38462
1.350	.57447	.42553	1.605	.61612	.38388
1.355	.57537	.42463	1.610	.61686	.38314
1.360	.57627	.42373	1.615	.61759	.38241
1.365	.57717	.42283	1.620	.61832	.38168
1.370	.57806	.42194	1.625	.61905	.38095
1.375	.57895	.42105	1.630	.61977	.38023
1.380	.57983	.42017	1.635	.62049	.37951
1.385	.58071	.41929	1.640	.62121	.37879
1.390	.58159	.41841	1.645	.62193	.37807
1.395	.58246	.41754	1.650	.62264	.37736
1.400	.58333	.41667	1.655	.62335	.37665
1.405	.58420	.41580	1.660	.62406	.37594
1.410	.58506	.41494	1.665	.62477	.37523
1.415	.58592	.41408	1.670	.62547	.37453
1.420	.58678	.41322	1.675	.62617	.37383
1.425	.58763	.41237	1.680	.62687	.37313
1.430	.58848	.41152	1.685	.62756	.37244
1.435	.58932	.41068	1.690	.62825	.37175
1.440	.59016	.40984	1.695	.62894	.47106
1.445	.59100	.40900	1.700	.62963	.37037
1.450	.59184	.40816	1.705	.63031	.36969
1.455	.59267	.40733	1.710	.63100	.36900
1.460	.59350	.40650	1.715	.63168	.36832
1.465	.59432	.40568	1.720	.63235	.36765
1.470	.59514	.40486	1.725	.63303	.36697
1.475	.59596	.40404	1.730	.63370	.36630
1.480	.59677	.40323	1.735	.63437	.36563
1.485	.59759	.40241	1.740	.63504	.36496
1.490	.59839	.40161	1.745	.63570	.36430
1.495	.59920	.40080	1.750	.63636	.36364
1.500	.60000	.40000	1.755	.63702	.36298
1.505	.60080	.39920	1.760	.63768	.36232
1.510	.60159	.39841	1.765	.63834	.36166
1.515	.60239	.39761	1.770	.63899	.36101
1.520	.60317	.39683	1.775	.63964	.36036
1.525	.60396	.39604	1.780	.64029	.35971
1.530	.60474	.39526	1.785	.64093	.35907

p/r	x	1 - x	p/r	x	1 - x
1.790	.64158	.35842	2.090	.67638	.32362
1.795	.64222	.35778	2.10	.67742	.32258
1.800	.64286	.35714	2.11	.67846	.32154
1.805	.64349	.35651	2.12	.67949	.32051
1.810	.64413	.35587	2.13	.68051	.31949
1.815	.64476	.35524	2.14	.68153	.31847
1.820	.64539	.35461	2.15	.68254	.31746
1.825	.64602	.35398	2.16	.68354	.31646
1.830	.64664	.35336	2.17	.68454	.31546
1.835	.64727	.35273	2.18	.68553	.31447
1.840	.64789	.35211	2.19	.68652	.31348
1.845	.64851	.35149	2.20	.68750	.31250
1.850	.64912	.35088	2.21	.68847	.31153
1.855	.64973	.35027	2.22	.68944	.31056
1.860	.65035	.34965	2.23	.69040	.30960
1.865	.65096	.34904	2.24	.69136	.30864
1.870	.65157	.34843	2.25	.69231	.30769
1.875	.65217	.34783	2.26	.69325	.30675
1.880	.65278	.34722	2.27	.69419	.30581
1.885	.65338	.34662	2.28	.69512	.30488
1.890	.65398	.34602	2.29	.69605	.30395
1.895	.65458	.34542	2.30	.69697	.30303
1.900	.65517	.34483	2.31	.69789	.30211
1.905	.65577	.34423	2.32	.69880	.30120
1.910	.65636	.34364	2.33	.69970	.30030
1.915	.65695	.34305	2.34	.70060	.29940
1.920	.65753	.34247	2.35	.70149	.29851
1.925	.65812	.34188	2.36	.70238	.29762
1.930	.65870	.34130	2.37	.70326	.29674
1.935	.65928	.34072	2.38	.70414	.29586
1.940	.65986	.34014	2.39	.70501	.29499
1.945	.66044	.33956	2.40	.70588	.29412
1.950	.66102	.33898	2.41	.70674	.29326
1.955	.66159	.33841	2.42	.70760	.29240
1.960	.66216	.33784	2.43	.70845	.29155
1.965	.66273	.33727	2.44	.70930	.29070
1.970	.66330	.33670	2.45	.71014	.28986
1.975	.66387	.33613	2.46	.71098	.28902
1.980	.66443	.33557	2.47	.71181	.28819
1.985	.66499	.33501	2.48	.71264	.28736
1.990	.66555	.33445	2.49	.71347	.28653
1.995	.66611	.33389	2.50	.71429	.28571
2.000	.66667	.33333	2.51	.71510	.28490
2.010	.66777	.33223	2.52	.71591	.28409
2.020	.66887	.33113	2.53	.71671	.28329
2.030	.66997	.33003	2.54	.71751	.28249
2.040	.67105	.32895	2.55	.71831	.28169
2.050	.67213	.32787	2.56	.71910	.28090
2.060	.67320	.32680	2.57	.71989	.28011
2.070	.67427	.32573	2.58	.72067	.27933
2.080	.67532	.32468	2.59	.72145	.27855

p/r	x	1 - x	p/r	x	1 - x
2.60	.72222	.27778	3.11	.75669	.24331
2.61	.72299	.27701	3.12	.75728	.24272
2.62	.72376	.27624	3.13	.75787	.24213
2.63	.72452	.27548	3.14	.75845	.24155
2.64	.72527	.27473	3.15	.75904	.24096
2.65	.72603	.27397	3.16	.75962	.24038
2.66	.72678	.27322	3.17	.76019	.23981
2.67	.72752	.27248	3.18	.76077	.23923
2.68	.72826	.27174	3.19	.76134	.23866
2.69	.72900	.27100	3.20	.76190	.23810
2.70	.72973	.27027	3.21	.76247	.23753
2.71	.73046	.26954	3.22	.76303	.23697
2.72	.73118	.26882	3.23	.76359	.23641
2.73	.73190	.26810	3.24	.76415	.23585
2.74	.73262	.26738	3.25	.76471	.23529
2.75	.73333	.26667	3.26	.76526	.23474
2.76	.73404	.26596	3.27	.76581	.23419
2.77	.73475	.26525	3.28	.76636	.23364
2.78	.73545	.26455	3.29	.76690	.23310
2.79	.73615	.26385	3.30	.76744	.23256
2.80	.73684	.26316	3.31	.76798	.23202
2.81	.73753	.26247	3.32	.76852	.23148
2.82	.73822	.26178	3.33	.76905	.23095
2.83	.73890	.26110	3.34	.76959	.23041
2.84	.73958	.26042	3.35	.77011	.22988
2.85	.74026	.26974	3.36	.77064	.22936
2.86	.74093	.25907	3.37	.77117	.22883
2.87	.74160	.25840	3.38	.77169	.22831
2.88	.74227	.25773	3.39	.77221	.22779
2.89	.74293	.25707	3.40	.77273	.22727
2.90	.74359	.25641	3.41	.77324	.22676
2.91	.74425	.25575	3.42	.77376	.22624
2.92	.74490	.25510	3.43	.77427	.22573
2.93	.74555	.25445	3.44	.77477	.22523
2.94	.74619	.25381	3.45	.77528	.22472
2.95	.74684	.25317	3.46	.77578	.22422
2.96	.74747	.25253	3.47	.77629	.22371
2.97	.74811	.25189	3.48	.77679	.22321
2.98	.74874	.25126	3.49	.77728	.22272
2.99	.74937	.25063	3.50	.77778	.22222
3.00	.75000	.25000	3.51	.77827	.22173
3.01	.75062	.24938	3.52	.77876	.22124
3.02	.75124	.24876	3.53	.77925	.22075
3.03	.75186	.24814	3.54	.77974	.22026
3.04	.75248	.24752	3.55	.78022	.21978
3.05	.75309	.24691	3.56	.78070	.21930
3.06	.75369	.24631	3.57	.78118	.21882
3.07	.75430	.24570	3.58	.78166	.21834
3.08	.75490	.24510	3.59	.78214	.21786
3.09	.75550	.24450	3.60	.78251	.21739
3.10	.75610	.24390	3.61	.78308	.21692

p/r	x	1 - x	p/r	x	1 - x
3.62	.78355	.21645	4.26	.80989	,19011
3.63	.78402	.21598	4.28	.81061	.18939
3.64	.78448	.21552	4.30	.81132	.18839
3.65	.78495	.21505	4.32	.81203	.18797
3.66	.78541	.21459	4.34	.81273	.18727
3.67	.78587	.21413	4.36	.81343	.18657
3.68	.78632	.21368	4.38	.81413	.18587
3.69	.78678	.21322	4.40	.81481	.18519
3.70	.78723	.21277	4.42	.81550	.18450
3.71	.78769	.21231	4.44	.81618	.18382
3.72	.78814	.21186	4.46	.81685	.18315
3.73	.78858	.21142	4.48	.81752	.18248
3.74	.78903	.21097	4.50	.81818	.18182
3.75	.78947	.21053	4.52	.81884	.18116
3.76	.78992	.21008	4.54	.81949	.18051
3.77	.79036	.20964	4.56	.82014	.17986
3.78	.79079	.20921	4.58	.82079	.17921
3.79	.79123	.20877	4.60	.82143	.17857
3.80	.79167	.20833	4.62	.82206	.17794
3.81	.79210	.20790	4.64	.82270	.17730
3.82	.79253	.20747	4.66	.82332	.17668
3.83	.79296	.20704	4.68	.82394	.17606
3.84	.79339	.20661	4.70	.82456	.17544
3.85	.79381	.20619	4.72	.82517	.17483
3.86	.77424	.20576	4.74	.82578	.17422
3.87	.79466	.20534	4.76	.82639	.17361
3.88	.79508	.20492	4.78	.82699	.17301
3.89	.79550	.20450	4.80	.82759	.17241
3.90	.79592	.20408	4.82	.82818	.17182
3.91	.79633	.20367	4.84	.82877	.17123
3.92	.79675	.20325	4.86	.82935	.17065
3.93	.79716	.20284	4.88	.82993	.17007
3.94	.79757	.20243	4.90	.83051	.16949
3.95	.79798	.20202	4.92	.83108	.16892
3.96	.79839	.20161	4.94	.83165	.16835
3.97	.79879	.20121	4.96	.83221	.16779
3.98	.79920	.20080	4.98	.83278	.16722
3.99	.79960	.20040	5.00	.83333	.16667
4.00	.80000	.20000	5.02	.83389	.16611
4.02	.80080	.19920	5.04	.83444	.16556
4.04	.80159	.19841	5.06	.83498	.16502
4.06	.80237	.19763	5.08	.83553	.16447
4.08	.80315	.19685	5.10	.83607	.16393
4.10	.80392	.19608	5.12	.83660	.16340
4.12	.80469	.19531	5.14	.83713	.16287
4.14	.80545	.19455	5.16	.83766	.16234
4.16	.80620	.19380	5.18	.83819	.16181
4.18	.80695	.19305	5.20	.83871	.16129
4.20	.80769	.19231	5.22	.83923	.16077
4.22	.80843	.19157	5.24	.83974	.16026
4.24	.80916	.19084	5.26	.84026	.15974

p/r	x	1 - x	p/r	x	1 - x
5.28	.84076	.15924	6.75	.87097	.12903
5.30	.84127	.15873	6.80	.87179	.12821
5.32	.84177	.15823	6.85	.87261	.12939
5.34	.84227	.15773	6.90	.87342	.12658
5.36	.84277	.15723	6.95	.87421	.12579
5.38	.84326	.15674	7.00	.87500	.12500
5.40	.84375	.15625	7.05	.87578	.12422
5.42	.84424	.15576	7.10	.87654	.12346
5.44	.84472	.15528	7.15	.87730	.12270
5.46	.84520	.15480	7.20	.87805	.12195
5.48	.84568	.15432	7.25	.87879	.12121
5.50	.84615	.15385	7.30	.87952	.12048
5.52	.84663	.15337	7.35	.88024	.11976
5.54	.84709	.15291	7.40	.88095	.11905
5.56	.84756	.15244	7.45	.88166	.11834
5.58	.84802	.15198	7.50	.88235	.11765
5.60	.84848	.15152	7.55	.88304	.11696
5.62	.84894	.15106	7.60	.88372	.11628
5.64	.84940	.15060	7.65	.88439	.11561
5.66	.84985	.15015	7.70	.88506	.11494
5.68	.85030	.14970	7.75	.88571	.11429
5.70	.85075	.14925	7.80	.88636	.11364
5.72	.85119	.14881	7.85	.88701	.11299
5.74	.85163	.14837	7.90	.88764	.11236
5.76	.85207	.14793	7.95	.88827	.11173
5.78	.85251	.14749	8.00	.88889	.11111
5.80	.85294	.14706	8.05	.88950	.11050
5.82	.85337	.14663	8.10	.89011	.10989
5.84	.85380	.14620	8.15	.89071	.10929
5.86	.85423	.14577	8.20	.89130	.10870
5.88	.85465	.14535	8.25	.89189	.10811
5.90	.85507	.14493	8.30	.89247	.10753
5.92	.85549	.14451	8.35	.89305	.10695
5.94	.85591	.14409	8.40	.89362	.10638
5.96	.85632	.14368	8.45	.89418	.10582
5.98	.85673	.14327	8.50	.89474	.10526
6.00	.85714	.14286	8.55	.89529	.10471
6.05	.85816	.14184	8.60	.89583	.10417
6.10	.85915	.14085	8.65	.89637	.10363
6.15	.86014	.13986	8.70	.89691	.10309
6.20	.86111	.13889	8.75	.89744	.10256
6.25	.86209	.13793	8.80	.89796	.10204
6.30	.86301	.13699	8.85	.89848	.10152
6.35	.86395	.13605	8.90	.89899	.10101
6.40	.86486	.13514	8.95	.89950	.10050
6.45	.86577	.13423	9.00	.90000	.10000
6.50	.86667	.13333	9.05	.90050	.09950
6.55	.86755	.13245	9.10	.90099	.09901
6.60	.86842	.13158	9.15	.90148	.09852
6.65	.86928	.13072	9.20	.90196	.09804
6.70	.87013	.12987	9.25	.90244	.09756

p/r	x	1 - x	p/r	x	1 - x
9.30	.90291	.09709	15.4	.93902	.06098
9.35	.90338	.09662	15.6	.93976	.06024
9.40	.90385	.09615	15.8	.94048	.05952
9.45	.90431	.09569	16.0	.94118	.05882
9.50	.90476	.09524	16.5	.94286	.05714
9.55	.90521	.09479	17.0	.94444	.05556
9.60	.90566	.09434	17.5	.94595	.05405
9.65	.90610	.09390	18.0	.94737	.05263
9.70	.90654	.09346	18.5	.94872	.05128
9.75	.90698	.09302	19.0	.95000	.05000
9.80	.90741	.09259	19.5	.95122	.04878
9.85	.90783	.09217	20.0	.95238	.04762
9.90	.90826	.09174	20.5	.95349	.04651
9.95	.90868	.09132	21.0	.95465	.04546
10.00	.90909	.09091	21.5	.95556	.04444
10.1	.90991	.09009	22.0	.95652	.04348
10.2	.91071	.08929	22.5	.95745	.04255
10.3	.91150	.08850	23.0	.95833	.04167
10.4	.91228	.08772	23.5	.95918	.04082
10.5	.91304	.08696	24.0	.96000	.04000
10.6	.91379	.08621	24.5	.96078	.03922
10.7	.91453	.08547	25.0	.96154	.03846
10.8	.91525	.08475	26	.96296	.03704
10.9	.91597	.08403	27	.96429	.03571
11.0	.91667	.08333	28	.96552	.03448
11.1	.91736	.08264	29	.96667	.03333
11.2	.91803	.08197	30	.96774	.03226
11.3	.91870	.08130	31	.96875	.03125
11.4	.91935	.08065	32	.96970	.03030
11.5	.92000	.08000	33	.97059	.02941
11.6	.92063	.07937	34	.97143	.02857
11.7	.92126	.07874	35	.97222	.02778
11.8	.92187	.07813	36	.97297	.02703
11.9	.92248	.07752	37	.97368	.02632
12.0	.92308	.07692	38	.97436	.02564
12.2	.92424	.07576	39	.97500	.02500
12.4	.92537	.07463	40	.97561	.02439
12.6	.92647	.07353	42	.97674	.02326
12.8	.92754	.07246	44	.97778	.02222
13.0	.92857	.07143	46	.97872	.02128
13.2	.92958	.07042	48	.97959	.02041
13.4	.93056	.06944	50	.98039	.01961
13.6	.93151	.06849	55	.98214	.01786
13.8	.93243	.06757	60	.98361	.01639
14.0	.93333	.06667	65	.98485	.01515
14.2	.93421	.06579	70	.98592	.01408
14.4	.93506	.06494	75	.98684	.01316
14.6	.93590	.06410	80	.98765	.01235
14.8	.93671	.06329	85	.98837	.01163
15.0	.93750	.06250	90	.98901	.01099
15.2	.93827	.06173	95	.98958	.01042
			100	.99010	.00990

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