

Fig. 7.13. Flow path in a 3-element hydrokinetic torque converter.

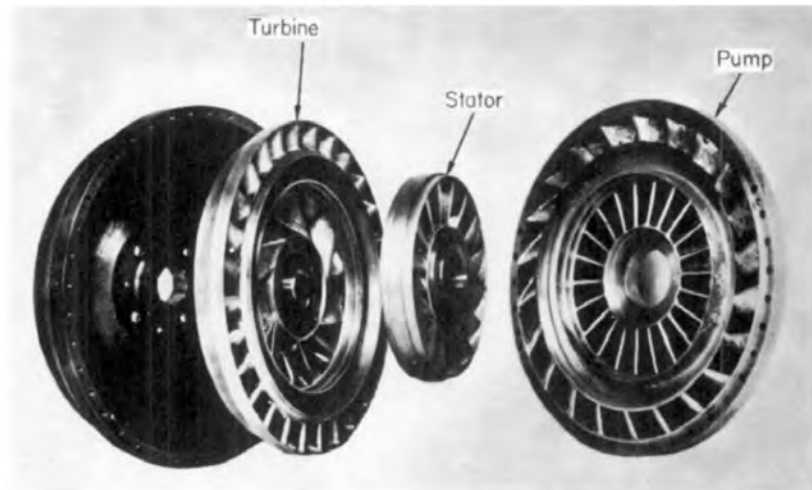


Fig. 7.12. Main components of a modern design of hydrokinetic torque converter coupling.

The Mechanics of Machines

The Mechanics of Machines

by

W. J. D. ANNAND

B.Sc., A.F.R.AE.S., A.M.I.MECH.E.

*Senior Lecturer, Department of Mechanical Engineering
University of Manchester*

CHEMICAL PUBLISHING CO., INC.
New York, N. Y.

The Mechanics of Machines

© 2012 Chemical Publishing Co., Inc. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Publisher, Chemical Publishing Company, through email at info@chemical-publishing.com.

The publisher and the author make no representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation warranties of fitness for a particular purpose.

ISBN: 978-0-8206-01205

First American Edition:

© **Chemical Publishing Company, Inc.** - New York 1966-2011

Second Impression:

Chemical Publishing Company, Inc. - 2012

Chemical Publishing Company:
www.chemical-publishing.com

Preface

When I began to teach the topics covered in this book to engineering undergraduates, I found great difficulty in suggesting suitable textbooks for study. There exist, on the one hand, several excellent books which treat mechanics analytically, more or less as applied mathematics; although the engineering student is perfectly capable of understanding these, in my experience he—being an engineer—usually finds that the formation of a mental picture of the physical events which are represented by the mathematics is essential to assimilation (as distinct from acceptance) of the theory. On the other hand, there are various books which tend to the encyclopaedic presentation of all the known methods of treatment of specific engineering problems; for the university and technical college student, these are unsuitable, because the essential principles are obscured by the mass of detail. The present book represents an attempt to fill the gap between these two classes, at an introductory level.

The content of the book is intended to provide a foundation of basic theory and an introduction to some important applications, such that the student should be made ready to begin, with a good physical grasp of essentials, the more advanced type of study appropriate to the final year of a first degree course. He is assumed to have initially a knowledge of physics and mathematics appropriate to university entrance—that is, at the Advanced Level of the General Certificate of Education. Some of the material he will have encountered already in his Advanced Level work, but he will almost certainly find it presented here in a different light. Whilst the book has been written with the university undergraduate student specifically in mind, it should be useful also in technical colleges and to those studying for the Graduate Membership examination of the Institution of Mechanical Engineers.

The first four chapters of the book deal with fundamentals, applicable to the analysis of any physical problem. The material contained in Chapter 1 does not always form part of the study of mechanics, but is in my opinion of considerable importance. Units are a constant source of unnecessary trouble to the engineering student; I have given

particular attention to force units. To the best of my knowledge, the content of Chapter 3 is novel as well as useful. Chapter 5 has more limited application, dealing rather with a fundamental technique. Chapters 6 to 8 cover various means of transmission of power, including hydrokinetic devices, information about which is elsewhere hard to find. Chapters 9 to 14 are devoted to a basic treatment of vibration theory; in a general book such as this it would be inappropriate (as well as impossible) to attempt an advanced study, which is better based upon one of the many excellent specialized texts cited, and the objective here has been to provide some idea of the nature of vibration problems and a general introduction to a few of the possible methods of attack upon them. A final chapter gives an introduction to the theory of control, illustrating in passing the close connection between control theory and vibration theory. Aiming essentially at physical clarity, I have in some places deliberately chosen methods of theoretical development which are not the most elegant. Where it seems appropriate, vector algebra is employed. This is done, partly for the sake of the greater clarity of representation of three-dimensional motion which the notation affords, and partly because I believe that familiarity with vector algebra is of great general value to the engineer; the topics treated here provide a useful illustration of its practical application at a very simple level. The elementary mathematical theory required is set out in a short appendix.

Each chapter concludes with a short bibliography, accompanied by a brief commentary on the fields of application of the publications cited. I hope that these may prove useful in the pursuit of information on particular problems; but it will also benefit the student greatly to read a few of the papers or books referred to, on the topics which he finds most interesting.

At the end of each chapter will be found also a number of exercises, some chosen from the past examination papers of various universities, and others prepared afresh for this book. They have been selected to illustrate the several points arising in the text, and in some cases to extend the text slightly. Duplication of problems of essentially the same nature has been avoided, and the student is strongly advised to work through all the exercises provided.

I have to thank, for permission to reproduce questions from past examination papers, the relevant officers of the Universities of Cambridge, Leeds, London, Manchester, Sheffield, and Wales (for the University College of Swansea); in doing so, I must emphasize that the solutions provided are my own, and are not endorsed by the examining bodies concerned.

Thanks are also due to Sir Isaac Pitman and Sons, Ltd, for per-

mission to adapt illustrations from *Gears* (3rd edn) by H. E. Merritt published by them in 1954; to the Institution of Mechanical Engineers for allowing the reproduction of figures 7.12 and 7.13 from *Proc. Automobile Division, I.Mech.E.*, London, 1956–57, 43; and to the editor of the *Bulletin of Mechanical Engineering Education* (Pergamon Press) for permission to re-use from an article published in 1964, (Vol. 3, p. 49) the illustrative example contained in Sections 5.16 and 5.17.

Finally, it gives me pleasure to thank the University of Manchester for facilities placed at my disposal; Professor J. Diamond, Beyer Professor of Mechanical Engineering at the University of Manchester, for his support and encouragement; and, of course, all my colleagues, who have helped to form my ideas.

W. J. D. Annand
July 1965

Contents

| | | |
|----|--|-----|
| 1 | Units and dimensions | 1 |
| 2 | Some consequences of Newton's laws of motion | 23 |
| 3 | Application of the first law of thermodynamics | 57 |
| 4 | Friction and lubrication | 68 |
| 5 | Plane linkage mechanisms | 91 |
| 6 | Power transmission by toothed gear wheels | 120 |
| 7 | Power transmission by hydrokinetic devices | 148 |
| 8 | Power transmission by belts | 171 |
| 9 | About vibration | 182 |
| 10 | Free vibration of undamped linear systems of one degree of freedom | 195 |
| 11 | Natural frequencies of higher-order linear systems | 220 |
| 12 | Vibration of a single-degree linear system with viscous damping and sinusoidal forcing | 240 |
| 13 | Vibration analogies | 262 |
| 14 | Sources of periodic exciting forces in machinery | 275 |
| 15 | Introduction to control theory | 303 |
| | Appendix Essentials of vector algebra | 333 |
| | Solutions to exercises | 347 |
| | Index | 351 |

I

Units and dimensions

1.1 Relevance of this chapter

No engineering calculation can be carried through without the substitution of numerical values for symbols in some equation. The question of what numerical values to insert involves consideration of units of measurement. Difficulties in the choice of numerical values often arise when the quantities to be handled include masses and forces, or when conversion between systems of units is required; difficulties which a grasp of a few simple ideas readily dispels.

Units of measurement are concrete quantities. We must also consider the abstract properties, called 'dimensions', which are measured. The relations between the dimensions of the quantities entering into any given engineering problem can be studied, and this study not only assists in clarifying the relations between units of measurement, but provides much guidance in approaching experimental investigation of complex problems. Indeed, without this guidance experimental work can, and frequently does, prove to be completely misdirected and valueless.

1.2 Units

Every measurement involves the comparison of some quantity, directly or indirectly, with a standard quantity of the same kind, which we call a 'unit of measurement'. When we measure a length, we commonly compare it directly with a standard length marked out on a scale—for example, a foot rule or a metre stick. For distances too long or too short to be measured conveniently by direct comparison with a physical standard, we may have to apply indirect

methods, such as those of surveying in the one case, or those of interferometry in the other, but the principle remains the same; the interpretation of the measurements finally rests upon comparison with some directly measured length. Some quantities—for example, mechanical power—can hardly be measured directly at all, but the idea is still preserved that there is a definable unit of power, and that comparison with this unit is the essence of measurement.

The size of the standard unit is essentially arbitrary. A yard, for example, is the distance between two marks on one particular bronze bar, at a specified temperature; all foot rules used in measurement are direct or indirect copies of one-third of that distance.

Between the units of some quantities there are obvious relations; thus a cubic foot, a unit of volume, is clearly related to the foot, which is a unit of length. In other cases, there may equally obviously be no relation—as for example, between a second (of time) and a square foot (of area). In yet other cases, it may not be immediately obvious whether there is, or is not, any relationship. The study of dimensions casts light on such questions.

1.3 Dimensions

A given length may be measured in inches, centimetres, or miles; all are 'units of length'. Two different lengths may not contain the same number of units, but they are obviously quantities of the same kind. What they share, extension, we call the 'dimension' of length. Similarly, two durations may contain different numbers of hours, seconds, or years, but they share the 'dimension' of time. The 'dimension' is the abstract property, the 'unit' the concrete measure of magnitude.

The dimensions of different kinds of quantity are, as will be seen, related, and it is possible to express the dimensions of all engineering quantities in terms of those of a chosen few. The relationships between dimensions tell us about the relationships between units, and have in addition other important uses.

Extension and duration are perhaps the simplest dimensional concepts that we have intuitively, and we usually take length and time among the 'fundamental' dimensions in terms of which others are to be expressed; but this is strictly a matter of choice, and we could choose others if we wished.

Another concept of which we have a rather vaguer intuitive idea is that of mass, which we regard as expressing the quantity of matter contained in a given body.

Dimensions may be represented symbolically, and can then be manipulated according to the ordinary rules of algebra. Thus if we write L for the dimension of length, we may write L^2 and L^3 for the dimensions of area and volume. Introducing T for the dimension of time, the dimension of velocity may be represented by L/T , that of acceleration by L/T^2 . We say that in terms of L and T, velocity 'has dimensions' L/T . Introducing M for the dimension of mass, density has dimensions M/L^3 ; and so on.

Now we find (as the remainder of this chapter will demonstrate) that for most mechanical and civil engineering problems it is sufficient to operate with three basic dimensions, and that those of length, mass, and time form a convenient trio.

Suppose that some quantity has, in these terms, the dimensions $M^a L^b T^c$. The ratio of two such quantities has, following the rules of algebraic manipulation, the dimensions

$$\frac{M^a L^b T^c}{M^a L^b T^c} = M^{a-a} L^{b-b} T^{c-c} = M^0 L^0 T^0 \quad (1.1)$$

We say that such a ratio is 'dimensionless', or a 'pure number'. Multiplication of any dimensional expression by $M^0 L^0 T^0$ must leave it unchanged—multiplication of any quantity by a pure number does not alter its dimensions. Accordingly, we might write the dimensions of the quantities discussed above in the following form, in order to keep before us that three basic dimensions are in use:

| QUANTITY | DIMENSIONS |
|--------------|--------------|
| Mass | ML^0T^0 |
| Length | M^0LT^0 |
| Time | M^0L^0T |
| Area | $M^0L^2T^0$ |
| Volume | $M^0L^3T^0$ |
| Velocity | M^0LT^{-1} |
| Acceleration | M^0LT^{-2} |
| Density | $ML^{-3}T^0$ |

It is not essential to use this form, but the student may find it helpful to do so until he becomes familiar with dimensional manipulation.

Angles are best thought of as fundamentally expressible in radian measure, that is, as the ratio of the length of any circular arc which subtends the given angle at its centre, to the radius of that arc. Then the dimensions in the M, L, T system are clearly $M^0L^0T^0$, so that angular velocity and angular acceleration have dimensions $M^0L^0T^{-1}$ and $M^0L^0T^{-2}$, respectively.

1.4 The choice of basic dimensions

It should be clearly understood that the choice of basic dimensions is essentially arbitrary, within the limitation that those selected must not be expressible entirely in terms of one another.

For example, we might (for some reason) choose area, velocity, and density. Writing A, V, and D for the dimensions of these, expressions for the others so far discussed could be found as follows:

| QUANTITY | DIMENSIONS |
|--------------------------------|--|
| Length = (Area) ^½ | $A^{½}V^0D^0$ |
| Volume = (Length) ³ | $(A^{½}V^0D^0)^3 = A^{3/2}V^0D^0$ |
| Time = Length/Velocity | $\frac{A^{½}V^0D^0}{A^0V^1D^0} = A^{½}V^{-1}D^0$ |
| Mass = Volume × Density | $(A^{3/2}V^0D^0)(A^0V^0D^1) = A^{3/2}V^0D^1$ |

and so on. These expressions are clumsier than those obtained in terms of M, L, and T and we may in addition ‘feel’ that mass, length, and time are more fundamental than the others, but all dimensional processes can be carried out equally well in either system.

1.5 Dimensional homogeneity

It is a fundamental principle that any valid relationship between physical quantities must be dimensionally homogeneous, which is to say that all the individual terms in any proposed relationship must have the same dimensions; we learn at school that we cannot add apples to eggs and get pence.

As a simple example, consider the equation for the velocity v attained by a body moving from rest with uniform acceleration f , after traversing a distance x

$$v = (2fx)^{½}$$

The dimensions are:

$$\begin{aligned} \text{Left-hand side: } & M^0L^1T^{-1} \\ \text{Right-hand side: } & (M^0L^1T^{-2} \cdot M^0L^1T^0)^{½} \\ & = (M^0L^2T^{-2})^{½} \\ & = M^0L^1T^{-1} \end{aligned}$$

so that this equation is dimensionally correct.

As an example of a widely-used equation which contains ‘concealed’ dimensions, consider the conventional equation for the volume rate

of flow Q over a rectangular-notch weir of notch width b under fluid head H .

$$Q = Kb(H)^{1.5}$$

where K is 'a constant'. Q has dimensions $M^0L^3T^{-1}$. Those of $bH^{1.5}$ are $M^0L^{2.5}T^0$. Thus the principle of homogeneity demands that K should have dimensions, which must be:

$$\frac{M^0L^3T^{-1}}{M^0L^{2.5}T^0} = M^0L^{\frac{1}{2}}T^{-1}$$

Two important conclusions follow:

1. K conceals some physical quantity, not expressly included in the equation. Unless we can determine what this quantity is, we ought to be sceptical about the general applicability of the equation, for it is reasonable to fear that the value of the concealed quantity might vary with the circumstances.
2. The numerical value of K will be different in different systems of units.

Clearly, it would be better to find out what the missing quantity is, and let it appear explicitly. The dimensional examination helps in this; the dimensions we have determined for K might be written $(M^0L^{\frac{1}{2}}T^{-2})^{\frac{1}{2}}$, the square root of those of acceleration, from which we might suspect (as is the case) that what is missing is the square root of the gravitational acceleration.

In engineering practice, we are often presented with empirical equations expressing experimentally-found relations between variables. It is well to subject these to dimensional examination. If they turn out to be inhomogeneous, as many are, caution is needed in applying them outside the range of the experimental data upon which they are based, and great care is also necessary to ensure that any constants occurring are correctly evaluated to suit the system of units in use.

1.6 Force

We obtain a general intuitive idea of force from our everyday manipulation of the objects around us, in terms of the pulls and pushes we have to exert. A sharper idea is needed, to allow quantitative use of the concept, and, for engineering purposes, we rely upon Newton's formulation. His first law is really a postulate, an unverifiable assumption; it states that every piece of matter remains stationary, or, if moving, continues to travel at constant velocity in a straight

line, except in so far as it is made by externally-applied forces to depart from that state. This effectively defines force by stating what happens in its absence, and implies that whenever a body accelerates a force must be acting upon it. The second law adds that, for a body of fixed mass, the force F applied is in the same direction as the acceleration and is numerically proportional to the product of the mass m and the acceleration f .

$$\begin{aligned} F &\propto mf \\ \text{or} \quad F &= k(mf) \end{aligned} \quad (1.2)$$

where k is a constant of proportionality. If we are to suppose that k is completely independent of the circumstances, then it must be a dimensionless number. The dimensions of F must, then, be the same as those of mf : in the M, L, T system, the dimensions of force are ML/T^2 .

1.7 Gravitation and weight

When Newton's laws were applied to planetary motion, it was necessary to postulate the existence of a force, acting between the heavenly bodies, in order to account for the fact that the planets move, not in straight lines, but along roughly elliptical paths. Calculation was found to agree with observation if it was assumed that any two particles of matter attract one another with a force, named 'gravitation', which is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them. A 'particle' is an infinitesimal piece of matter; to find the attraction exerted by a large body, the forces produced by all the particles of which it is composed must be summed. In this way, it can be shown that a spherical body of uniform density has the same effect as would be produced if all its mass were concentrated at its centre.

Accordingly, a large sphere of mass M attracts a small body of mass m , placed outside the sphere at a radius r from its centre, with a gravitational force (GMm/r^2), where G is a 'universal gravitational constant'. If no other force acts upon the small body, it must accelerate towards the centre of the sphere at a rate f given by

$$\frac{GMm}{r^2} = kmf \quad (1.3)$$

$$\text{or} \quad f = \frac{GM}{k} \left(\frac{1}{r^2} \right)$$

so that the acceleration is independent of the mass of the small body.

Although the earth is not a true sphere, and is not of uniform density, the force experienced by a small body placed near its surface does not differ greatly from that given by the above equation; let us say that the exact value is $c(GMm/r^2)$, where c is a correction factor very nearly equal to unity. If the body rotates with the earth, the force needed to support it at a fixed distance above the surface will in general be a little less than $c(GMm/r^2)$, because a small resultant force is needed to keep the body in its circular path; the difference varies from zero at the poles to a maximum of about 0.4% at the equator.

The force which a body so supported applies to its support is what we call its 'weight'. It is what we measure by suspending the body from a suitably calibrated spring balance. The weight of a given piece of material varies from place to place on the earth's surface, partly because of the centrifugal effect just noted, partly because the earth's radius is less at the poles than at the equator, and partly because of local surface irregularities such as mountain chains.

At a place where a body of mass m has weight W , it will, if not supported nor acted upon by any other force, accelerate, approximately towards the centre of the earth, at a rate which is, as before, independent of the mass. This acceleration is called (slightly in-exactly) the 'local gravitational acceleration'. It is denoted by g , and we have:

$$W = kmg \quad (1.4)$$

1.8 The choice of a unit of force

Given that the sizes of units of mass, length, and time have been defined, the choice of a value for k in the Newtonian equation settles the size of the unit of force.

To recapitulate, we have for the force required to give acceleration f to mass m (equation 1.2)

$$F = kmf$$

and for the weight of a body of mass m at a place on the earth's surface where the local gravitational acceleration is g (equation 1.4)

$$W = kmg$$

If, as in this book, we are interested mainly in the 'dynamic' forces associated with the acceleration of bodies, it is convenient to set $k = 1$, so that unit force gives to unit mass an acceleration of one unit. The resulting group of force-mass-length-time units is called

an 'absolute' or 'logical' system (although there is nothing absolute, and very little logical, about any actual system).

If, on the other hand, we are concerned mainly with weight forces—'dead loads'—imposed upon fixed structures, it is convenient to choose a force unit approximately equal to the local weight of unit mass. We then want k to be approximately equal numerically to $1/g$; to avoid the inconvenience of a variable unit, we select an average numerical value of g , denoted by g_0 , and set $k = 1/g_0$. The local weight of a body of mass m , in these units, is then

$$(1/g_0)mg \text{ force units}$$

1.9 Units of force associated with the pound mass, foot, and second

The 'logical' unit of force in terms of the pound mass, foot, and second is that which gives an acceleration of 1 ft/sec^2 to a mass of 1 pound. This unit is called a 'poundal', but it is sometimes useful to remember that it could equally well be called a 'pound ft/sec²'.

To give to a mass of m pounds an acceleration $f \text{ ft/sec}^2$ requires a force of mf poundals.

The weight of a mass of m lb at a place where the gravitational acceleration is $g \text{ ft/sec}^2$ is mg poundals.

In this book, we shall work almost entirely in the poundal–pound–mass–foot–second system. However, the engineer must be familiar with the 'non-logical' system which is (regrettable as it may be) much more widely used in Britain and North America. This is of the 'dead load' type, with g_0 set equal to 32.1741 (an internationally accepted standard) so that the unit of force is the weight of 1 pound mass at a place where the local gravitational acceleration is 32.1741 ft/sec^2 . This unit is called a 'pound force'.

Accordingly, to give to a mass of m pounds an acceleration $f \text{ ft/sec}^2$ requires a force of $(1/g_0)mf$ pounds force, and the weight of a body of mass m is $(1/g_0)mg$ pounds force.

Obviously, g_0 is the conversion factor from pounds force to poundals: 1 pound force = g_0 poundals.

The use of the pound-force–pound-mass–foot–second system involves difficulties of two types:

1. Uncertainty as to the need to insert g_0 in the evaluation of equations expressing complicated dynamic problems. This is commonly called 'g trouble'. It is best avoided, by working always in poundals, converting from and to pound force units as required.

2. Confusion over the names of derived units, resulting from the use of the name 'pound' for two different entities. Quotation of the gas constant in units described as 'pounds foot per pound degree temperature' is a regrettably common example; one is tempted to suppose that this might be simplified to feet per degree, but of course the first pound is force and the second is mass. To avoid this we must always write 'pound force', and not just 'pound', when appropriate.

We shall adopt the British Standard convention of writing the contraction lb for pound mass and lbf for pound force.

1.10 Other systems

In aerodynamics, it has become the British and North American practice to adopt a 'logical' system based on the foot and second but using the pound force as a unit. To give $k = 1$, the mass unit has to contain g_0 pounds mass, and this is called one 'slug'.

The metric system provides, of course, the 'logical' centimetre-gram-second (c.g.s.) system generally used in physics. The force unit of 1 dyne gives to a mass of 1 gram an acceleration of 1 cm/sec². The dyne being very small, engineers have developed what is called the metre-kilogram-second (m.k.s.) system. In this, the mass unit is the kilogram, and the force which gives one kilogram an acceleration of one metre per second per second is called 1 'newton'. One newton-metre per second equals 1 watt, so the m.k.s. system is widely used among electrical engineers in Britain and North America as well as in the countries using the metric system.

Even in these countries, a 'dead-weight' system of units is used by many engineers. In that system, g_0 is 981, and the weight of 1 kilogram mass at a place where $g = g_0$ cm/sec² is called one kilogram force.

All these force-mass systems may be summarized as follows:

| SYSTEM | FORCE | MASS UNIT | ACCELERATION | k |
|---------------------|---------|-----------|---------------------|-----------|
| | UNIT | | UNIT | |
| British engineering | lbf | lb | ft/sec ² | 1/32.1741 |
| British MLT | poundal | lb | ft/sec ² | 1 |
| British FLT | lbf | slug | ft/sec ² | 1 |
| Metric engineering | kgf | kg | cm/sec ² | 1/981 |
| C.G.S. | dyne | g | cm/sec ² | 1 |
| M.K.S. | newton | kg | m/sec ² | 1 |

1.11 Energy, work, and power

The kinetic energy of a mass m moving at velocity v is $\frac{1}{2}mv^2$. The dimensions of kinetic energy can, therefore, be written ML^2/T^2 , and, using the relation given above, this is equivalent to FL. Now the energy stored in lifting a body against the gravitational force, which is a form of potential energy, is equal to the product of the force and the height through which the body is lifted, and this clearly also has dimensions FL or ML^2/T^2 . We are accustomed to the idea that various forms of energy can be interchanged, and the equality of dimensions of these two forms reflects this.

In terms of units, it will be apparent that $1 \text{ lb ft}^2/\text{sec}^2$ and 1 pdl ft are equivalent names for the same amount of energy, and that 1 lbf ft contains $g_0 \text{ pdl ft}$.

'Work' is the name we give to energy when it is being transferred mechanically from one 'system' to another, just as 'heat' is the name we give to energy transferred thermally. It is essentially a transient thing; a machine may 'contain' energy but we cannot say that it 'contains' work, only that it has a certain capacity to do work when called upon. If a force applied to any object moves it some distance in the direction of the force, the work done = force \times distance. The dimensions FL are those of energy, as they should be. If a torque applied to a shaft rotates it, the work done is the product of the torque and the radian measure of the angle turned through. The dimensions of torque are FL and the angle is 'dimensionless', which again gives the correct dimensions for the work done.

'Power' is the rate at which work is done, and its dimensions are therefore FL/T or ML^2/T^3 . The engineering unit of power used in Britain and North America is the horsepower, usually abbreviated hp, equivalent to 33,000 ft lbf/min. The corresponding unit used on the Continent of Europe, the Cheval Vapeur or Pferd Stärke, is rather smaller; being 75 m kgf/sec, it amounts to 0.9863 hp.

1.12 Dimensions and units of two properties of fluids

Two properties of fluids which commonly enter into engineering considerations are viscosity and surface tension. Viscosity is the measure of the resistance of a fluid to shearing motion; if a fluid flows slowly along a plane surface, the ratio of the tangential force experienced by each unit area of the surface to the velocity gradient normal to the surface (see figure 1.1) is the viscosity. Thus its

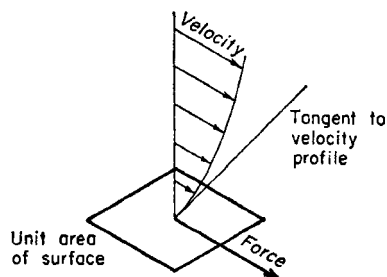


Fig. 1.1. Defining viscosity.

dimensions are those of (force per unit area) divided by (velocity per unit distance), i.e.

$$\left(\frac{ML/T^2}{L^2}\right) \div \left(\frac{L/T}{L}\right) = \frac{ML \cdot TL}{T^2 L^2 \cdot L} = \frac{M}{LT}$$

Suitable units are, accordingly, lb/ft sec and 1 lb/ft sec is the same thing as 1 pdl sec/ft². The larger unit 1 lbf sec/ft² contains g_0 of these. In the metric system, 1 dyne sec/cm², which is the same as 1 g/cm sec, is called 1 poise.

Surface tension is an effect of the cohesive force between the molecules of a liquid, producing the appearance at any surface that the liquid is enclosed in an elastic film. The strength of this 'film' is measured by the tensile force which a surface of unit width can sustain without rupture, and this is called the surface tension. The dimensions are $(ML/T^2) \div L$ or M/T^2 . The units mainly used are dyne/cm or lbf/ft.

$$1 \text{ lbf/ft} = g_0 \text{ pdl/ft} = g_0 \text{ lb/sec}^2$$

1.13 Thermal and electrical quantities

We shall not deal here to any extent with thermal or electrical quantities other than the simplest, and they will, therefore, be only briefly commented upon. It is possible to say, for example, that heat is a form of energy transfer and, therefore, has dimensions ML^2/T^2 and that temperature can be thought of as a measure of heat absorption per unit mass, so that it has dimensions L^2/T^2 ; and similar devices can be used to reduce electrical quantities to terms of 'mechanical' ones. It is ordinarily more useful, however, to regard temperature in the one case and electric charge in the other as

Index

- ACCELERATION, CORIOLIS, 107**
of points on a rigid link, 104, 106
relative, 95
Acceleration meter, 256
Addendum, gear tooth, 124
Amonton's laws of friction, 69
Angle, dimensions of, 3
- BALANCE OF ROTATING SHAFT, 275**
Bearing, ball, 87
dry, 79
hydrodynamic, 80
hydrostatic, 85
Belt drive, arrangement, 171
effect of mass, 175
efficiency, 178
tensioning, 176
torque capacity, 173
Bevel gears, 122
Bladed rotor, action of, 149
Bode diagram, 316
Boundary lubrication, 77
- CLAMPED JOINTS, 74**
Coefficient of restitution, 34
Collisions, maximum force during, 35
rotational, 48
translational, 33
Conjugate profiles, gears, 125
Conservative force, 37
Control, open and closed loops, 304
problems of, 304
Control systems, diagrams, 305
response, 308
stability, 328
transfer operators, 309
vector response diagrams, 319
Control volume, 57
Coriolis acceleration, 107
Coulomb friction, 69
Creep in belt drives, 178
Cross-product of inertia, 40
- DAMPING, 190**
Damped vibration, caused by shaken support, 254
characteristics, 248
dynamic magnifier, 246, 258
steady forced, 246
transient, 242
transmissibility, 249
Dedendum, gear tooth, 124
Degrees of freedom, vibrational, 188
Dimension, definition, 2
symbolic representation, 3
Dimensional analysis, application to modelling, 18
basis, 13
limitations, 17
manipulation of groups, 17
method, 14
short cuts, 18
Dimensional homogeneity, 4
Dimensionless quantity, 3
Dimensions, choice of basic, 4
Displacement meter, 256
- ENERGY, DIMENSIONS AND UNITS OF, 10**
internal, 58
rate of change of, inertial, 61
strain, 63
Euler's equations of motion, 42
- FEEDBACK, 305**
First law of thermodynamics, 58
applied, 111, 142
Fluid coupling, description, 150
efficiency, 152
heat dissipation, 161
partial filling, 154
selection, 158
uses, 156
Flywheel, 45
Force, definition, 5
dimensions of, 6

- Force, units of, 7
 Four bar chain, 93
 accelerations in, 105
 velocities in, 98
 Fretting, 73
 Friction, coefficient of, definition, 69
 values, 72
 dry sliding, 68
 effects of, in first law equation, 64
 maintenance of vibration by, 75
- GEAR PAIR TYPES, 121**
 Gear pitch, 124
 Gear tooth forms, geometrical limitations, 128
 involute, 126
 Novicov, 127
 practical considerations, 129
 Gear tooth numbers, 133
 Gear trains, compound, 132
 driving torque, 142
 epicyclic, bevel, 141
 compound, 138
 multiple, 139
 simple, 136
 simple, 130
 Geared shafts, sudden connection, 48
 vibration, 214
 Gravitation, 6
 Gyroscopic couple, 50
- HELICAL GEARS, 121**
 Helmholtz resonator, 262
 Hypoid gears, 123
 Hysteresis in belts, 178
- IMPACT, *see* COLLISIONS**
 Impulse, rotational, 43
 translational, 33
 Involute gear, 126
- KINEMATIC ANALYSIS, METHODS OF, 93**
 Kinematics, 91
- LINEAR SPRINGS, 207**
 combination, 208, 210
 Link, 91
 Linkage mechanisms, forces in, 113
 Lubrication, boundary, 77
 hydrodynamic, 78
 hydrostatic, 85
- MECHANISM, 91**
 Mitre gears, 122
 Module, 124
 Moment of inertia, definition, 40
 of geared system, 43, 63, 143
 Moment of momentum, 39
 Momentum, definition, 23
 of a rigid body, 27
 Momentum equation, 29
- NATURAL FREQUENCY, GEARED SYSTEM, 214**
 importance of, 192
 methods of estimation, energy, 199
 201, 202
 Holzer, 228
 node-fixing, 212, 220
 Rayleigh, 235
 receptance, 203, 226
 s.h.m., 198
 uniform beam, 232
 Newton's Laws, 23
 Novicov gear, 127
- PERIODIC MOTION, 182**
 Petroff's equation, 87
 Pinion, 121
 Pitch of gear, 124
 Pitch point, 124
 Polyphase torque converter, 165
 Position control, definition, 303
 simple, 307
 viscous friction in, 311
 Potential energy, 37
 Power, dimensions and units of, 10
 Pressure angle, 126
 Principal axes, 41
- QUICK-RETURN MECHANISM, ACCELERATIONS in, 109**
 velocities in, 102
- RACK, 121**
 Ratio, dimensions of, 3
 Receptance, direct, definition, 203
 graphical representation, 207
 of mass, 204
 of spring, 205
 of system, 205, 224, 225
 indirect, 223
 Reciprocating engine forces, 282
 harmonic components, 286

- in-line engine, 288
- radial engine, 300
- vector representation, 287
- vee engine, 295
- pitching couples, 291
- Reference frame, rotating, 24
- Regulator, 303
- Response locus, 320
- Response of second-order system,
 - to harmonic input, 315
 - to ramp input, 323
 - to step input, 313
 - vector representation, 319
 - with derivative terms, 324
 - with integral terms, 327
- Rigid body, definition, 26
 - non-rotation conditions, 28
 - rotational motion, 39
- Ring gear, 121
- Rocket, 29
- Rope drive, 175
- SELF-EXCITED VIBRATION, 185
- Silencer, 265
- Similarity, dynamic, 18
- Simple harmonic motion, 195
- Simple pendulum, 198
- Slider-crank chain, 93
- Sliding connections, relative accelerations, 107
 - relative velocities, 101
- Slip in fluid coupling, 152
- Sommerfeld number, 83
- Spiral gears, 122
- Spur gears, 121
- Stall, in fluid coupling, 152
 - in torque converter, 163
- Surface tension, dimensions and units of, 11
- THERMODYNAMIC SYSTEM, 57
- Thermodynamics, relevance of, 57
- Torque capacity of fluid coupling, 153
- Torque coefficient of fluid coupling, 152
- Torque converter, hydrokinetic,
 - characteristics, 163
 - description, 161
 - polyphase, 164
 - selection, 164, 166
 - uses, 166
- Torque, vector representation of, 339
- Transmission of vibration, 249
- UNITS OF MEASUREMENT, 1
 - conversion, 12
- VECTOR, DEFINITION, 333
 - differentiation, 342
 - modulus, 335
 - scalar product, 337
 - triple products, 342
 - vector product, 339
- Vee belt, 175
- Velocity, of points on a rigid link, 98
 - measurement, 256
 - relative, 94
- Vibration analogues, 262
- Vibration, degrees of freedom, 188
 - free and forced, 184
 - frequency, 184
 - importance of, 185
 - linearity, 189
 - modes of, 187
 - period, 184
- Viscosity, dimensions and units of, 11
- WEIGHT, 7
- Work, dimensions and units of, 10
- Work-energy equation, 32, 43
- Worm gear, 123

